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# 8

## **FUNDAMENTAL ASPECTS OF GASEOUS BREAKDOWN-I**

**E**lectrical breakdown in gases has been studied extensively for over one hundred years and the delineation of the various manifestations of discharges has advanced in parallel with a better understanding of the fundamental processes. This vast research area is covered by several excellent books<sup>1, 2</sup>. Fig. 8.1 shows different types of discharge one encounters in practice depending upon the combinations of parameters.

The type of discharge is determined by primary factors such as gas pressure, gas density, electrode shape and distance, polarity of the voltage, type of voltage meaning dc, normal frequency ac, high frequency ac, impulse voltage, etc. Secondary factors are the electrode materials, type and distance to the enclosure, duration of the application of voltage, previous history of the electrodes, etc. Obviously it is not intended to explain all of these phenomena even in a condensed fashion but the fundamental processes that occur are similar, though the intensity of each process and its contribution to the overall discharge process varies over a wide range. In the interest of clarity we limit ourselves to fundamental processes concentrating on the progress that has been achieved during the past twenty five years. However, to provide continuity we recapitulate some fundamental definitions and equations that are relevant to all discharge processes.

### **8.1 COLLISION PHENOMENA**

#### **8.1.1 ELASTIC COLLISION**

An electron acquires energy in an electric field and during its acceleration elastic collision occurs between an electron and a molecule. During an elastic collision there is very little exchange of energy with the electron, losing an energy that is proportional to  $m/M$  where  $m$  and  $M$  are masses of the electron and molecule, respectively. The internal

energy levels of the molecule do not change during an elastic collision. The main consequence of an elastic collision is that the direction of travel of an electron changes depending upon the angle at which the electron strikes the molecule. A more accurate term for elastic collisions is the **momentum transfer collision** which is an average value that takes into account the angle of approach of the electron.

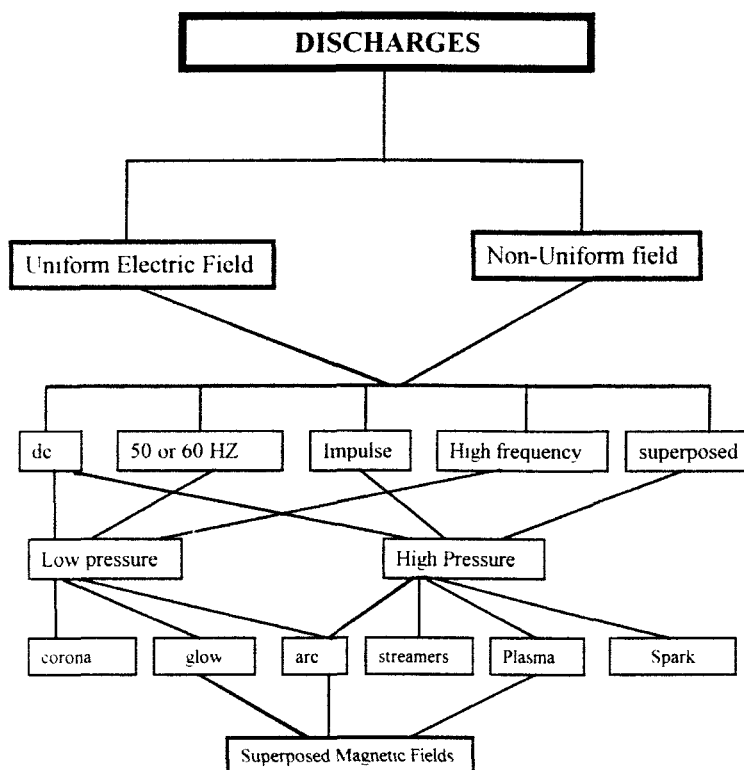


Fig. 8.1 Various manifestations of electrical discharges.

The range of electron energy and the electron density encountered in a wide range of plasmas are shown in fig. 8.2<sup>3</sup>. The parameters chosen to characterize the plasmas are the electron temperature expressed in units of electronvolts and the electron density. One electronvolt (eV) is equal to 11600 degree Kelvin in accordance with  $T = (e/k) \epsilon$  where

$\epsilon$  = energy in eV

$e$  = electronic charge

$k$  = Boltzmann constant

The energy at 300 K is correspondingly 0.026 eV. The electron energy shown in Fig. 8.1 varies over eight decades while the electron density spans a wide range of twenty decades depending upon the type of plasma. Interplanetary space has the lowest number

density whereas the fusion plasma has the highest. Ionosphere plasma has the lowest temperature with fraction of an electronvolt energy. Fusion reactors, on the other hand, have an energy  $\sim 100,000$  eV. Such a wide range of parameters makes the study of plasmas one of the most interesting.

### 8.1.2 COLLISION CROSS SECTION

The collision between two particles is described by using a fundamental property called the collision cross section, which is defined as the area involved between the colliding particles measured in  $\text{m}^2$ . Collisions between neutral particles, or between a neutral particle and a charged particle, may be considered to a first approximation as if the particles were hard spheres. The effective target area for such collisions is the collision cross section,  $Q$ , which has a value of  $10^{-19} \text{m}^2$  in air for gas molecules. If each particle has a radius  $a$  then the collision cross section is given by  $4\pi a^2$ . Each inelastic process is characterized by a corresponding cross section and the total cross section is the sum of the momentum transfer and inelastic collision cross sections.

### 8.1.3 PROBABILITY OF COLLISION

The probability of collision is defined as the average number of collisions that an electron makes with a neutral molecule per meter length of drift. The probability is related to the collision cross section according to

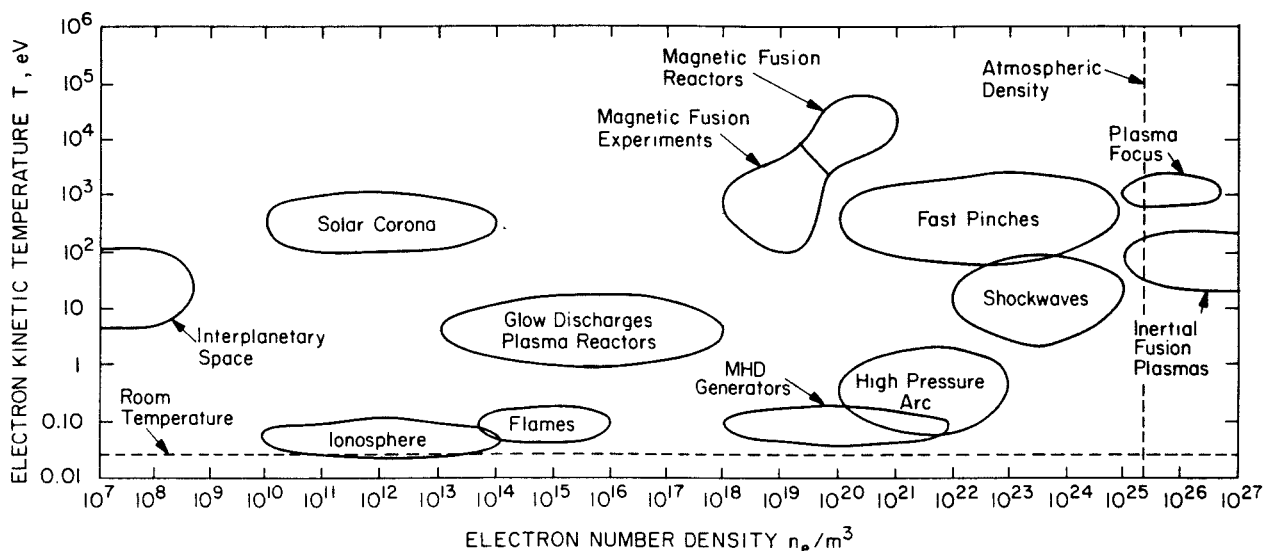


Fig. 8.2 Electron number density and electron energy in various plasmas (Roth, 1995, with permission of Institute of Physics.)

$$P = N Q_m \text{ where}$$

$Q_m$  = Momentum transfer cross section ( $m^2$ )

$P$  = Number of collisions ( $m^{-1}$ )

$N$  = Number density of molecules ( $m^{-3}$ ). The number density is related to gas pressure at 300 K according to the following relationships.

$$N = 3.220 \times 10^{22} \times p \text{ (Torr)} \quad (m^{-3})$$

$$N = 2.415 \times 10^{20} \times p \text{ (Pa)} \quad (m^{-3})$$

$$N = 2.447 \times 10^{25} \times p \text{ (atm)} \quad (m^{-3})$$

$$1 \text{ atmosphere} = 760 \text{ Torr} = 101.33 \text{ kPa}$$

The number of collisions,  $N_c$ , per second is given by

$$\text{Number of collisions per second} = N Q_m v$$

$v$  = velocity ( $m/s$ )

The probability of collision for electrons in gases is a function of electron energy and is usually plotted against  $\sqrt{\epsilon}$  where  $\epsilon$  is the electron energy.

For electrons having energy greater than about 1 eV,  $P_c$  increases with  $\epsilon$  reaching a maximum in the range of 10-15 eV. For a further increase in energy the probability decreases at various rates depending on the gas (Fig. 8.3). At energies lower than 1 eV the probability of collision is a complicated function due to quantum mechanical effects. In many gases the quantum mechanical wave diffraction of the electron around the atom results in an increased probability with decreasing energy, as found experimentally by Ramsauer during early thirties.

Compilation of cross sections for elastic scattering and momentum transfer for common gases is available in ref.<sup>4</sup>. The cross sections for carbon compounds such as  $CH_4$ ,  $CF_4$ ,  $CF_2Cl_2$  etc. which are of interest in plasma processing applications are compiled in ref.<sup>5</sup>.

### **8.1.4 INELASTIC COLLISIONS**

Electrons gain energy from the applied electric field and at sufficiently high energy a collision will result in a change of the internal energy level of the molecule. Such a collision is called an inelastic collision. Inelastic collisions with atoms or molecules mainly result in creation of excited species, ionization and metastables (excited

molecules with a long life time). Some common inelastic collisions are given in Table 8.1.

### **8.1.5 MEAN FREE PATH**

The **mean free path** of an electron,  $\lambda$ , is the average distance traveled by an electron between collisions with gas molecules. The concept of the mean free path is general and not limited to elastic collisions. We can refer to a mean free path for elastic collisions as distinct from mean free path for ionizing collisions,  $\lambda_i$ . We can also refer to a mean free path for collisions between gas atoms or molecules even in the absence of an electric field. The mean free path for molecules at atmospheric pressure of 101 kPa is approximately 60 nm, a very small distance. As the pressure decreases the mean free path increases and the relationship is

$$\lambda = \frac{1}{P} = \frac{1}{NQ_m} \quad (8.1)$$

**Table 8.1**

Some commonly observed inelastic collisions

| Type                     | Reaction                          |
|--------------------------|-----------------------------------|
| Ionization               | $e + x \rightarrow x^+ + e + e$   |
| Excitation               | $e + x \rightarrow x^* + e$       |
| Dissociative attachment  | $e + x y \rightarrow x^- + y + e$ |
| Dissociation             | $e + x y \rightarrow x + y + e$   |
| Recombination            | $x^+ + y^- \rightarrow x + y$     |
| Three body recombination | $e + x^+ + y^- \rightarrow x + y$ |

The distance traveled between collisions is a varying quantity due to the random nature of collisions. The probability of having a free path greater than  $x$  is an exponentially decaying function according to  $\exp(-x/\lambda)$  where  $\lambda$  is the mean free path.

### **8.1.6 IONIZATION BY COLLISION**

When the energy of the electron exceeds the ionization potential of a gas molecule a secondary electron and a positive ion are formed following a collision. The ionization cross section of a electron,  $Q_i$ , increases with its energy reaching a peak value at about

three times the ionization energy. For a further increase the ionization cross section decreases slowly. Two basic mechanisms for ionization are:

### 8.1.7 DIRECT IONIZATION

A molecule is directly impacted by an electron of sufficient energy

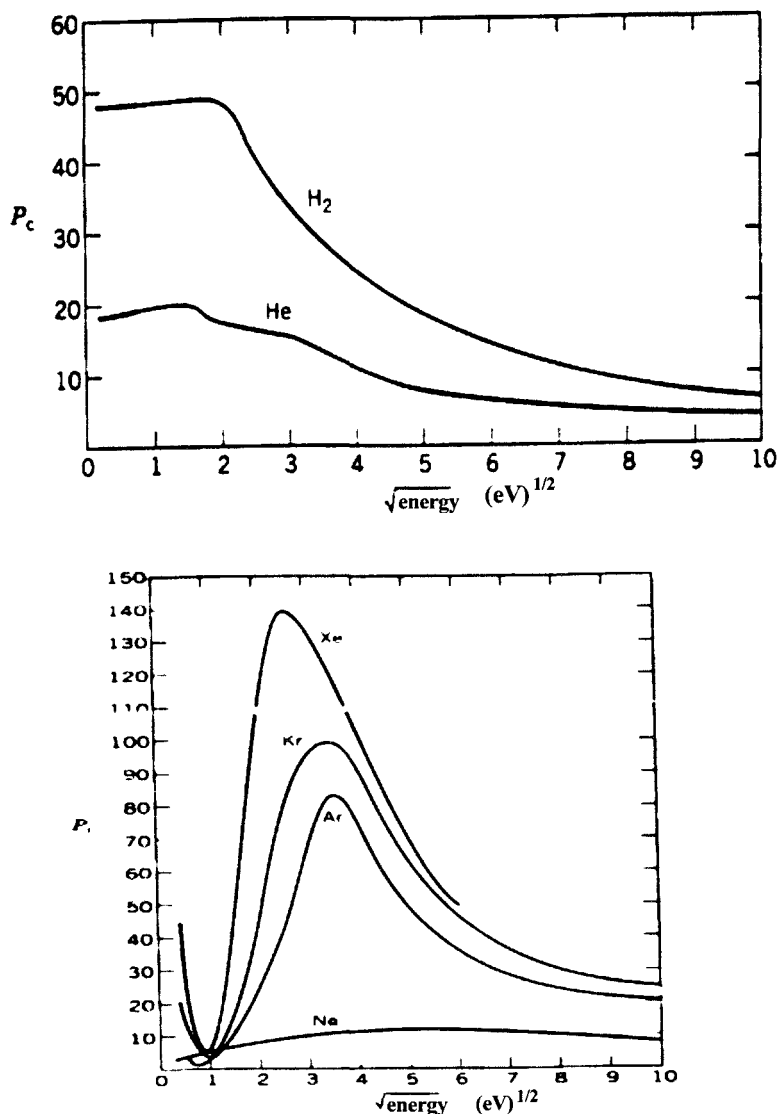
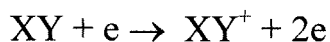
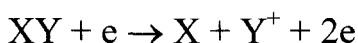


Fig. 8.3 Probability of collision for electrons in gases. Lower figure shows the Ramsauer minima at low energies. The cross section is given by  $Q_m = 2.87 \times 10^{-21} P_c \text{ m}^2$ .

### **8.1.8 DISSOCIATIVE IONIZATION**



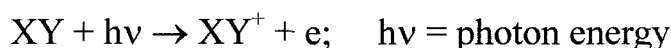
X, Y gas atoms or molecules

$Y^+$  = positive ion

e = electron.

Total ionization cross sections for rare and common molecular gases are available in ref.<sup>6, 7</sup>. Cross sections for dissociative ionization for several molecular gases are reported in ref.<sup>8</sup>. Figure 8.4 shows the ionization cross sections as a function of electron energy for several molecular gases<sup>9</sup>.

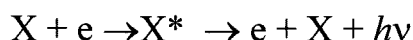
Photo-ionization occurs when photons of energy greater than the ionization potential of the molecule,  $\epsilon_i$  impinge on the molecule. This reaction is represented by:



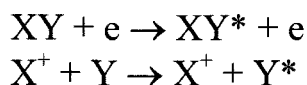
### **8.1.9 EXCITATION**

The first excitation threshold of an atom is lower than the ionization potential and the excitation cross section is generally higher than the ionization cross section. The excited species returns to its ground state after a short interval, ~10 ns emitting a photon of equivalent energy.

The direct excitation mechanism is



Where  $X^*$  denotes an excited atom. Two other types are



where  $X^+$  is a positive ion.

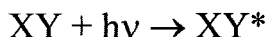


### **8.1.10 DISSOCIATIVE EXCITATION**

The electron impact dissociates the molecule and the excess energy excites one of the atoms



### **8.1.11 PHOTOEXCITATION**

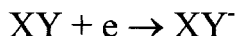


A compilation of excitation cross sections can be found in ref.<sup>10</sup>.

### **8.1.12 ELECTRON ATTACHMENT**

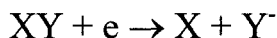
Atoms which are electronegative have an affinity for an electron and the process of an electron being captured by such an atom is called attachment. Molecules having an electron attaching atom as a constituent also become electron attaching. Oxygen and halogens are electron attaching elements. Examples of electron attaching molecules are: O<sub>2</sub>, CO, CO<sub>2</sub>, SF<sub>6</sub> etc. Several processes occur:

#### **A. DIRECT ATTACHMENT**



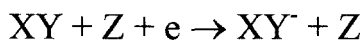
XY<sup>-</sup> = Negative ion

#### **B. DISSOCIATIVE ATTACHMENT**



#### **C. THREE BODY ATTACHMENT**

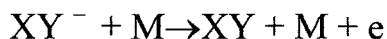
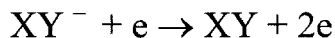
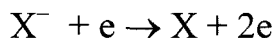
Three body attachment occurs in the presence of a molecule that stabilizes and promotes charge transfer



Total cross sections for negative ion formation in several gases (CO, NO, O<sub>2</sub>, CO<sub>2</sub> and SF<sub>6</sub>) by electron impact are given in Rapp and Braglia (1963)<sup>11</sup>.

### 8.1.13 ELECTRON DETACHMENT

Electrons may be detached from the negative ions by processes



The last process is known as three-body attachment and if active, the detachment coefficient is pressure dependent. Electron detachment increases the population of electrons which may further participate in the ionization process.

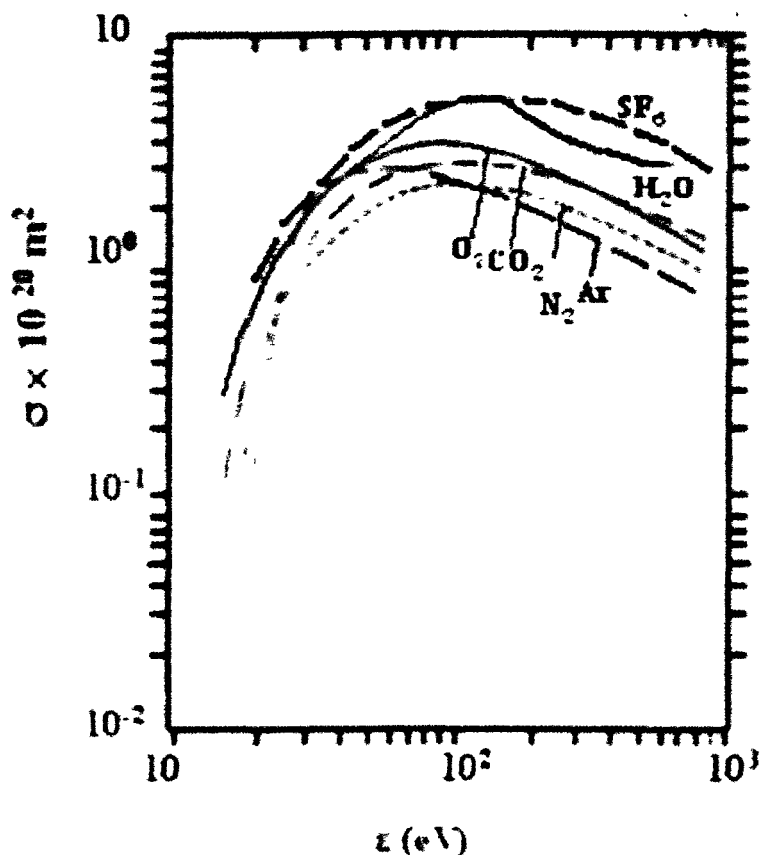
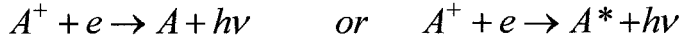


Fig. 8.4 Ionization cross sections for several molecules as a function of electron energy (Pejcev et. al., 1979, with permission of American Chemical Society).

### 8.1.14 RECOMBINATION

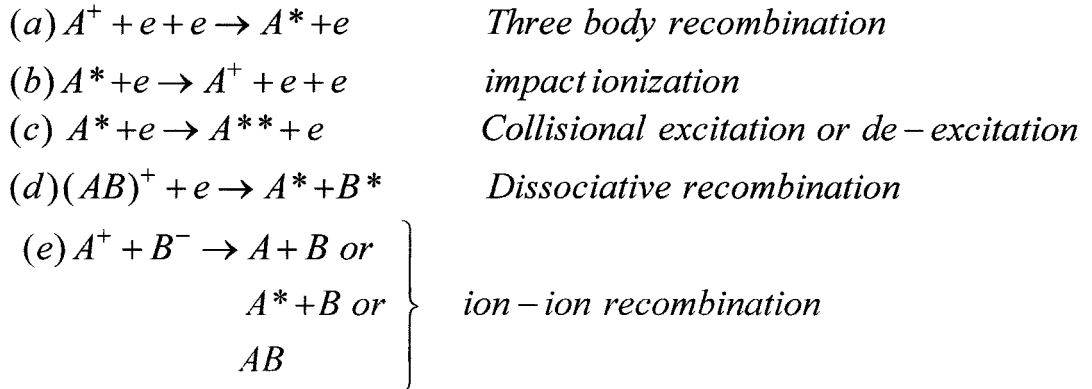
The recombination of positive and negative charges occurs through several different mechanisms. The simplest reaction is



These are two body radiative processes and the rate of loss of electrons is given by

$$\frac{\partial n_e}{\partial t} = -\alpha_e n_e^2 \quad (8.2)$$

where  $n_e$  is the density of electrons assumed to be equal to the density of positive ions and  $\alpha_e$  is the recombination coefficient ( $\text{cm}^3 \text{ s}^{-1}$ ). It has values in the range  $1 \times 10^{-14} \leq \alpha_e \leq 1 \times 10^{-7} \text{ cm}^3 \text{ s}^{-1}$  depending on the gas, electron density and temperature. Increasing number density and temperature yields higher values of  $\alpha_e$ . Several other processes are possible (Meek and Craggs, 1978)



In considering recombination many of these processes have to be taken into account acting in combination and the decrease of electron density due to recombination has led to the term **collisional radiative decay**. At low electron densities,  $10^7 \leq n_e \leq 10^{12} \text{ cm}^{-3}$ , and low electron temperatures  $\sim 1 \text{ eV}$  the recombination coefficient is relatively constant, independent of  $n_e$ . In this region two body recombination processes are dominant. As the temperature of electrons increase the recombination coefficient increases with  $n_e$ . This situation arises in considering recombination in hot spark channels, and three body processes become increasingly important.

### **8.1.15 SECONDARY IONIZATION COEFFICIENT**

There are several mechanisms which yield secondary electrons both at the cathode and in the gas. The secondary electrons contribute to the faster growth of current in the discharge gap and the important mechanisms of secondary ionization are classified as below.

#### **A. Impact of Positive Ions on the Cathode**

Positive ions which impinge on the cathode liberate secondary electrons provided their energy is equal to or higher than the work function of the cathode. The number of secondary electrons liberated per incident ion is dependent on the surface conditions of the cathode such as oxidation, adsorbed gas layer, etc. The secondary emission is higher for cleaner and non-oxidized surfaces. At low gas pressures the secondary ionization coefficient is usually of the order of  $10^{-2} - 10^{-4}$  per incident electron. In discharges at higher pressures ( $\sim 100$  kPa.) the influence of secondary electrons due to positive ions is negligible as the ions do not have sufficient energy.

#### **B. Impact of Metastables on the Cathode**

Under certain conditions an excited molecule loses a fraction of its energy by collision and the result is a new excited state from which the excited molecule cannot return to the ground state. The life time of such excited molecules, called metastables, is about  $10^{-3}$  s. Some metastables are destroyed by collision with a gas molecule and some are lost by falling on the anode. However, secondary emission can occur when a metastable strikes the cathode. For a given gas in which the metastables are present (the rare gases and nitrogen are examples) the secondary emission due to positive ions will be more pronounced than that due to metastables. The reason is that, under any particular condition, the positive ions are attracted to the cathode due to Coloumb force where as the metastables, being charge neutral, can only reach the cathode by diffusion. Further, the probability of liberation of a secondary electron due to positive ions is higher than for metastables.

#### **C. Photoelectric Action**

Emission of electrons from the cathode due to photoelectric action is an important secondary mechanism and is effective at moderate gas pressures ( $\sim 10$ - $100$  kPa) and gap lengths of a few centimeters. A copious supply of photons of sufficient wave length and a low absorption coefficient in the gas render this secondary process more effective. All of the photons generated will not fall on the cathode since most of them are created near

the anode and the number decreases exponentially with gap distance due to photon absorption according to

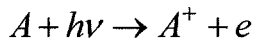
$$n = n_o \exp(-\mu x) \quad (8.3)$$

where  $n$  is the number of photons that survive after a transit distance of  $x$  from the avalanche head (anode),  $n_o$ , the initial number of number of photons generated and  $\mu$  the absorption co-efficient.  $\mu$  is a function of wave length and measurements of absorption co-efficients in gases using monochromatic beams have been published. However these data cannot be used in gas discharge studies because the discharge produces photons having various energies. Govinda Raju et al<sup>12</sup> have measured photon absorption in several gases using a self sustained discharge as source of photons. A corona discharge in a wire-cylinder geometry was also employed<sup>13</sup> and absorption coefficient of  $0.4 \text{ cm}^{-1}$  at gas number density of  $\sim 10^{17} \text{ cm}^{-3}$  was measured. This amounts to a decrease of photon intensity of  $\sim 33\%$  for a transit of 1 cm and the reduction will be greater at higher gas pressures or longer gap lengths.

The secondary ionization co-efficient observed experimentally is a combination of several of these effects and the relative contributions have been determined for several gases. Generally speaking at low gas number densities (higher E/N) the positive ion action predominates whereas at high gas pressures,  $\sim 100 \text{ kPa}$ , the photoelectric action predominates. The secondary ionization coefficient also increases with E/N as shown in Fig. 8.5<sup>14</sup>.

### **8.1.16 PHOTO-IONIZATION**

Photons having energy equal to or greater than the ionization potential of the gas cause photo-ionization and the process is defined as



This process is the converse of radiative electron-ion recombination. Photo-ionization has been invoked in the streamer mechanism (section 8.3) as the principal mechanism in the transition of a primary avalanche into a streamer in gases at atmospheric pressure. The photo-ionization cross sections for a number of gases have been measured by using line spectrum (mono-energy) rather than a continuum. The cross section for this process for several molecules are given in Table 8.2.

While these photo-ionization cross sections are of interest from the point of view atomic and molecular physics, it is not possible to apply them directly to electrical discharges because the discharge produces photons of various energies. The relative intensity of photons generated is also not known. Further, in a strongly photon absorbing gas, the photons generated in the discharge disappear at a very short distance from the detector and are not available at a distance for measurement. Due to these difficulties photon absorption experiments have been carried out at low gas pressures and the results are then applied to high density discharges. Table 8.3 shows the absorption coefficient for photo-ionizing radiations in several gases.

The intensity of photons scattered in a gas is attenuated according to

$$I = I_o \exp(-\mu_p x)$$

where  $\mu_p$  is the absorption coefficient ( $\text{m}^{-1}$ ) and  $x$  is the distance (m). The absorption coefficient is related to the photo-ionization cross section according to

$$\mu_p = NQ_v$$

where  $N$  is the gas number density and  $Q_v$  the photo-ionization cross section.

### **8.1.17 ELECTRON SWARM COEFFICIENTS**

The coefficients listed in Table 8.4 are usually referred to as swarm parameters in the literature on gas discharges.

Consider a swarm of electrons or ions moving through a reduced electric field  $E/N$  and having a drift velocity of  $W$  m/s. The mobility is defined as  $\mu = W/E$  and in the literature on gas discharges mobility is often referred to standard conditions of pressure and temperature ( $p = 760$  torr and  $T = 273$  K) by expressing

$$\mu_o = W \frac{P_{273}(\text{Pa})}{E} \frac{1}{101.3 \times 10^7} \frac{273}{T(K)} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \quad (8.4)$$

where  $\mu_o$  is called the reduced mobility.

In terms of E/N we have the relationship

$$\mu_o = W \frac{N}{E} \frac{1}{101.3 \times 10^7} \frac{1}{3.54 \times 10^{16}} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \quad (8.5)$$

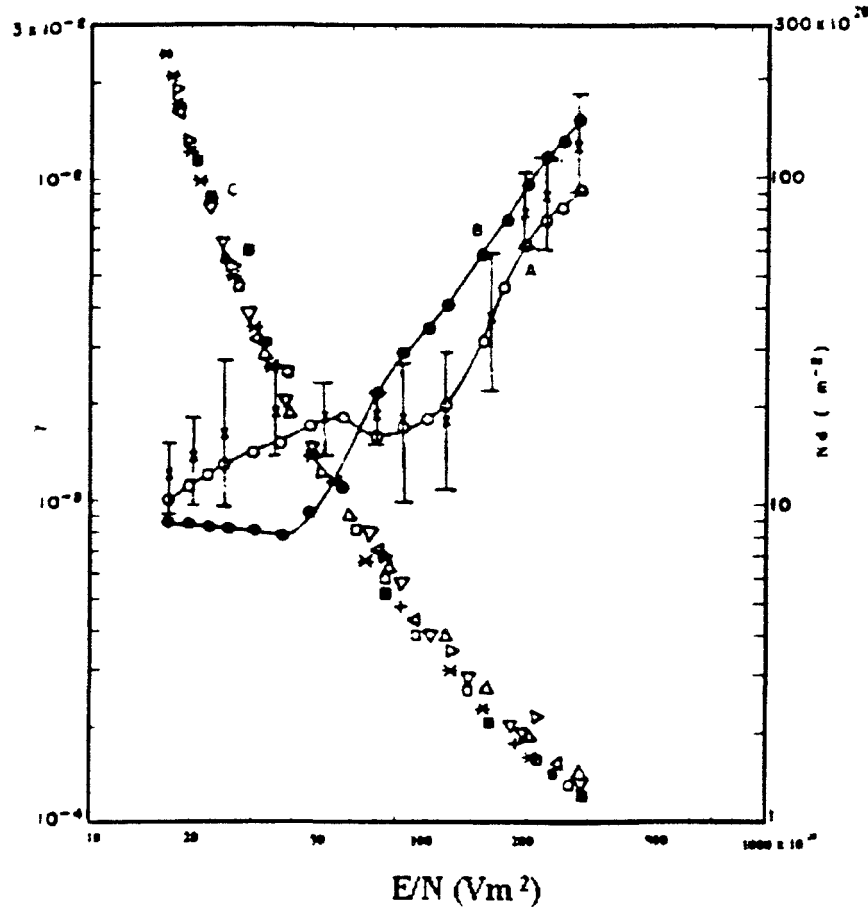


Fig. 8.5  $\gamma$  as a function of E/N for air and a gold plated cathode before glow discharge (curve A) and after glow discharge (curve B). The vertical bars indicate the spread of values obtained from  $\log(I, N)$  plots. The plot of E/N against Nd is included to show the value of N corresponding to each value of E/N (curve c), (Raju and Rao, 1971, with permission of Institute of Physics, UK.)

Experimental and theoretical values for the swarm parameters for a large number of gases are available in the literature. Table 8.5 shows the data for gases which are of practical interest. The swarm coefficients are expressed as a function of the parameter E/N, called **reduced electric field**, which has the dimension of  $\text{V m}^{-2}$ . In older literature the reduced electric field is E/p with the dimension of  $\text{V cm}^{-1} \text{ Torr}^{-1}$ , the conversion factor being,  $E/p \times 3.1 \times 10^{-21} = E/N (\text{V m}^{-2})$ , 1 Townsend (Td) =  $1 \times 10^{-21} \text{ V m}^{-2}$ .

**Table 8.2**

Photo-ionization cross sections (Meek &amp; Craggs, 1978).

| Molecule       | Wave length (nm) | Cross section ( $\times 10^{-18} \text{ cm}^2$ ) |
|----------------|------------------|--------------------------------------------------|
|                | 101.9 $\pm$ 1 nm | onset                                            |
| Oxygen         | 97.9             | 7.04 $\pm$ 1.8                                   |
|                | 47.3             | 23.1 $\pm$ 4.1                                   |
| Nitrogen       | 78.7             | 8.8 $\pm$ 1.6                                    |
|                | 47.3             | 20.8 $\pm$ 3.8                                   |
| Carbon dioxide | 88.3 $\pm$ 1.5   | onset                                            |
|                | 70               | 30                                               |
|                | 98.5 $\pm$ 0.5   | onset                                            |
| Water vapor    | 65               | 20                                               |
|                | 47               | 12                                               |
| Hydrogen       | 80.37            | onset                                            |
|                | 77.0-66.5        | 8                                                |
| Argon          | 79.0 $\pm$ 0.5   | onset                                            |
|                | 77.0             | 27                                               |

(With permission of John Wiley &amp; Sons)

**Table 8.3**Photon absorption coefficients<sup>15</sup>

| Gas                              | N ( $\times 10^{16} \text{ cm}^{-3}$ ) | $\mu^{**} (\text{cm}^{-1})$ | Photon efficiency* | source             |
|----------------------------------|----------------------------------------|-----------------------------|--------------------|--------------------|
| O <sub>2</sub>                   | 32.2-96.6                              | 38                          | $\sim 10^{-3}$     | Cylindrical corona |
|                                  | 3-10                                   | $\sim 250$                  |                    | "                  |
|                                  | 1-3                                    | $\sim 550$                  |                    | "                  |
|                                  | 40-220                                 | 2.5                         | $\sim 10^{-5}$     | spark              |
|                                  | 10-30                                  | 38                          | $\sim 10^{-2}$     | "                  |
| N <sub>2</sub>                   | 1-5                                    | $\sim 750$                  | $\sim 10^{-3}$     | Cylindrical corona |
| Air                              | 10-200                                 | $\sim 5$                    | $2 \times 10^{-3}$ | "                  |
| CO <sub>2</sub>                  | 1-3                                    | 200-800                     | $\sim 10^{-4}$     | "                  |
| CH <sub>4</sub>                  |                                        | 970                         |                    | "                  |
| C <sub>2</sub> H <sub>5</sub> OH |                                        | 400-700                     |                    | "                  |
| H <sub>2</sub> O                 |                                        | >200                        |                    | "                  |

\*per ionizing collision, \*\* Normalized to atmospheric pressure.

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**Table 8.4**  
Electron Swarm Parameters

| Parameter                      | Symbol              | Units                      |
|--------------------------------|---------------------|----------------------------|
| Drift velocity                 | $W$                 | $\text{m s}^{-1}$          |
| Mean energy                    | $\bar{\varepsilon}$ | eV                         |
| Characteristic energy          | $D/\mu$             | eV                         |
| Reduced ionization coefficient | $\alpha/N$          | $\text{m}^2$               |
| Reduced attachment coefficient | $\eta/N$            | $\text{m}^2$               |
| Reduced detachment coefficient | $\delta/N$          | $\text{m}^2$               |
| Recombination coefficient      | $\beta$             | $\text{m}^3 \text{s}^{-1}$ |

Electron swarm parameters are an indispensable set of data for each gas to understand the discharge mechanisms and to develop theoretical approaches. A number of these parameters in analytical form for several gases is provided below.

**Table 8.5**  
Swarm Parameters in Gases

**Air<sup>16</sup>**

Ionization coefficient

$$\frac{\alpha}{N} \begin{cases} = 2.0 \times 10^{-20} \exp\left[(-7.248 \times 10^{-19} \times \frac{N}{E})\right] & \text{m}^2; E/N > 1.5 \times 10^{-19} \text{Vm}^2 \\ = 6.619 \times 10^{-21} \times \exp\left[(-5.593 \times 10^{-19} \times \frac{N}{E})\right] & \text{m}^2; E/N \leq 1.5 \times 10^{-19} \text{Vm}^2 \end{cases}$$

Attachment coefficient

$$\frac{\eta_1}{N} = 4.33 \times 10^{-4} \times \frac{E}{N} - 1.0 \times 10^{-23} \text{m}^2$$

Three body attachment coefficient

$$\frac{\eta_2}{N^2} = 4.778 \times 10^{-69} \times \left(\frac{E}{N} \times 10^4\right)^{-1.2749} \text{m}^5$$

$$\eta = \eta_1 + \eta_2$$

### Recombination coefficient

$$\beta = 2.0 \times 10^{-13} \text{ m}^3 \text{ s}^{-1}$$

### **A. Oxygen<sup>17</sup>**

Electron drift velocity:  $W_e = 5.747 \times 10^{16} (E/N)^{0.0604} \text{ ms}^{-1}$

Positive and negative ion mobility:  $\mu_i = 5.5 \times 10^{21} (1/N) \text{ V}^{-1} \text{ s}^{-1} \text{ m}^2$

Recombination coefficient:  $\beta = 2.0 \times 10^{-13} \text{ m}^3 \text{ s}^{-1}$

### Electron diffusion coefficient

$$\frac{D}{\mu} = 1.343 \times 10^6 (E/N)^{0.3441} \text{ V}; E/N < 2.0 \times 10^{-21} \text{ Vm}^2$$

$$\frac{D}{\mu} = 1.213 \times 10^{25} (E/N)^{1.2601} \text{ V}; 2 \times 10^{-21} < E/N < 1.23 \times 10^{-20} \text{ Vm}^2$$

$$\frac{D}{\mu} = 1.519 \times 10^9 (E/N)^{0.46113} \text{ V}; E/N > 1.23 \times 10^{-20} \text{ Vm}^2$$

### Two body attachment coefficient

$$\frac{\eta_1}{N} = 2.0 \times 10^{-20} \text{ m}^2; E/N > 1.835 \times 10^{-20} \text{ Vm}^2$$

$$\frac{\eta_1}{N} = 5.032 \times 10^{-30} (E/N)^{2.655} \text{ m}^2; E/N < 1.835 \times 10^{-20} \text{ Vm}^2$$

### Three body attachment rate

$$K_a = \left[ \frac{3.0 \times 10^{-42}}{1 + \left( \frac{E}{N} \frac{1}{4.0 \times 10^{-21}} \right)^{3/2}} \right] \text{m}^6 \text{s}^{-1}$$

### Three body attachment coefficient

$$\frac{\eta_2}{N} = \frac{NK_a}{W_e} \text{m}^2$$

Total attachment coefficient:  $\eta = \eta_1 + \eta_2$

### Ionization coefficient

$$\frac{\alpha}{N} = 2.124 \times 10^{-20} \exp(-6.2116 \times 10^{-19} \frac{N}{E}) \text{m}^2$$

## **C. Sulphur hexafluoride**<sup>18</sup>

### Electron drift velocity

$$W_e = 1.027 \times 10^{19} (E/N)^{0.7424} \text{ms}^{-1}$$

### Positive ion mobility (reduced)

$$\mu_{0(+)} = 6.0 \times 10^{-5} \text{m}^2 \text{V}^{-1} \text{s}^{-1}; E/N < 1.2 \times 10^{-19} \text{Vm}^2$$

$$\mu_{0(+)} = 1.216 \times 10^{-5} \ln(E/N) + 5.89 \times 10^{-4} \text{m}^2 \text{V}^{-1} \text{s}^{-1}; \\ 1.2 \times 10^{-19} < E/N < 3.5 \times 10^{-19} \text{Vm}^2$$

$$\mu_{0(+)} = -1.897 \times 10^{-5} \ln(E/N) - 7.346 \times 10^{-4} \text{m}^2 \text{V}^{-1} \text{s}^{-1}; E/N > 3.35 \times 10^{-19} \text{Vm}^2$$

### Negative ion mobility (reduced)

$$\mu_{0(-)} = 1.69 \times 10^{-32} (E/N)^2 + 5.3 \times 10^{-5} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}; E/N < 5.0 \times 10^{-19} \text{ V m}^2$$

The actual mobility is given by

$$\mu = \mu_0 \frac{p_0}{p} \frac{T}{T_0}$$

where  $p_0 = 101.325 \text{ kPa}$  and  $T_0 = 273.16 \text{ K}$ .

### Recombination coefficient

$$\beta = 2.0 \times 10^{-13} (P)^{0.6336} \text{ m}^3 \text{ s}^{-1}; \quad 1 \leq p \leq 39 \text{ kPa}$$

$$\beta = 2.28 \times 10^{-11} (P)^{-0.659} \text{ m}^3 \text{ s}^{-1}; \quad 39 \leq P \leq 270 \text{ kPa}$$

$$\beta = 6.867 \times 10^{-10} (P)^{-1.279} \text{ m}^3 \text{ s}^{-1}; \quad 270 \leq P \leq 2000 \text{ kPa}$$

### Negative ion diffusion coefficient

$$\frac{D}{\mu} = \frac{k_T}{39.6} \text{ V}$$

where  $k_T$ , the Townsend's energy factor is given by Table 8.6.

**Table 8.6**

Mean values of  $k_T$  and  $D/\mu$  for negative ions in  $\text{SF}_6$  (Morrow, © 1986, IEEE)

|             |       |       |       |       |       |       |       |
|-------------|-------|-------|-------|-------|-------|-------|-------|
| E/N (Td)    | 30.3  | 60.7  | 91.0  | 121.4 | 151.7 | 182   | 212.3 |
| $k_T$       | 1.0   | 1.2   | 1.7   | 2.6   | 3.5   | 4.6   | 5.9   |
| $D/\mu$ (V) | 0.025 | 0.030 | 0.043 | 0.066 | 0.086 | 0.116 | 0.149 |

### Electron diffusion (longitudinal)

$$\frac{D_L}{\mu} = 8.6488 \times 10^9 (E/N)^{1/2} \text{ V}; E/N < 6.5 \times 10^{-19} \text{ V m}^2$$

Attachment coefficient: (Fig. 8. 6)<sup>19</sup>

$$\frac{\eta}{N} = 2.0463 \times 10^{-20} - 0.25379(E/N) + 1.4705 \times 10^{18}(E/N)^2 - 3.0078 \times 10^{36}(E/N)^3 \text{ m}^2;$$
$$5.0 \times 10^{-20} < E/N < 2.0 \times 10^{19} \quad \text{Vm}^2$$
$$= 7.0 \times 10^{-21} \exp(-2.25 \times 10^{18} \times E/N) \quad \text{m}^2; \quad E/N > 2.0 \times 10^{19} \quad \text{Vm}^2$$

Ionization coefficient (Fig. 8.6) (Raju and Liu, 1995)

$$\frac{\alpha}{N} = 3.4473 \times 10^{34}(E/N)^{2.985} \quad \text{m}^2; \quad E/N < 4.6 \times 10^{-19} \quad \text{Vm}^2$$
$$= 11.269(E/N)^{1.159} \quad \text{m}^2; \quad E/N > 4.6 \times 10^{-19} \quad \text{Vm}^2$$

Table 8.7 summarizes the investigations on  $\alpha$  and  $\eta$  in SF<sub>6</sub>.

## **8.2 ELECTRON GROWTH IN AN AVALANCHE**

Electrons released from the cathode due to an external source of irradiation such as ultraviolet light multiply in the presence of ionization provided the applied electric field is sufficiently high. The multiplication depends on the parameter E/N according to

$$n = n_0 e^{\alpha d} \tag{8.6}$$

$n_0$  = number of initial electrons

$\alpha$  = Townsend's first ionization coefficient (m<sup>-1</sup>)

$d$  = gap length (m)

Equation (8.6) is generally adequate at gas pressures greater than about 15 kPa. At lower gas pressures the electron has to travel a minimum distance before it acquires adequate energy for the onset of the ionization process and equation (8.6) then becomes

$$n = n_0 \exp(d - d_0) \tag{8.7}$$

$d_0$  = minimum distance for equilibrium (m).

Equations (8.6) and (8.7) apply when the electron multiplication is not large enough to generate secondary electrons. The sources for secondary electrons are photoelectric action and positive ion action at the cathode. The secondary ionization co-efficient is defined as the number of secondary electrons generated per primary electron and usually denoted by the symbol  $\gamma$ . The contribution due to photoelectric action and positive ion impact are denoted by  $\gamma_{ph}$  and  $\gamma_+$  so that  $\gamma = \gamma_{ph} + \gamma_+$ . The current growth in the presence of secondary ionization is expressed as

$$i = i_0 \frac{\exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]} \quad (8.8)$$

The electron growth is also influenced by attachment and detachment processes. The attachment coefficient,  $\eta$ , ( $m^{-1}$ ), is defined analogous to  $\alpha$  as the number of electrons lost due to a primary electron during a drift of unit distance.

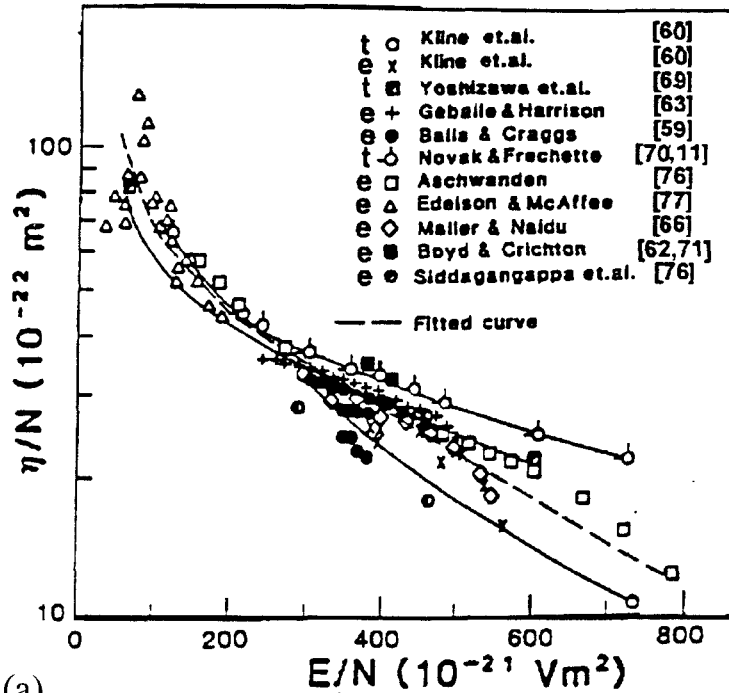
The current growth in an electron attaching gas occurs according to

$$i = i_0 \frac{\alpha \exp\{(\alpha - \eta)d\} - \eta}{(\alpha - \eta)} \quad (8.9)$$

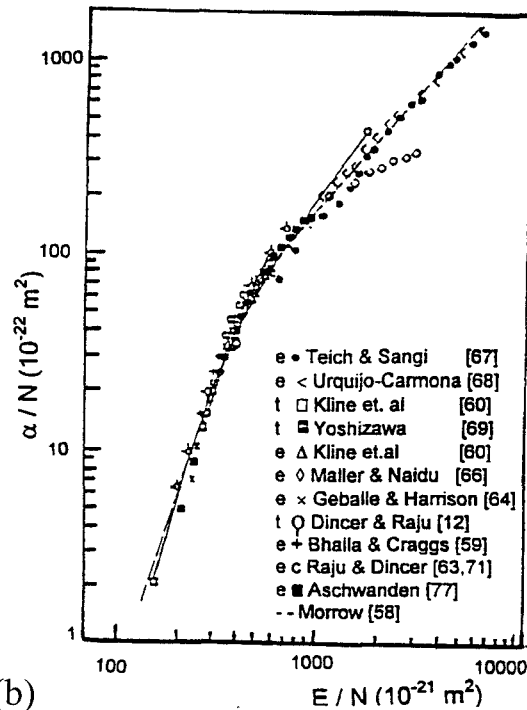
The current growth shown in equation (8.9) assumes that the secondary processes are not active. To take into account the secondary electrons released from the cathode the current growth equation should be modified as:

$$i = i_0 \frac{\left( \frac{\alpha}{\alpha - \eta} \exp(\alpha - \eta)d - \frac{\eta}{\alpha - \eta} \right)}{1 - \left\{ \frac{\gamma \alpha}{(\alpha - \eta)} [\exp(\alpha d) - 1] \right\}} \quad (8.10)$$

Electron detachment is also a process that occurs in some electron attaching gases such as oxygen and  $SF_6$ . denoting the detachment coefficient as  $\delta$  an effective attachment co-efficient  $\eta^*$  may be defined as  $\eta^* = (\eta - \delta)$  in moderately detaching gases such as  $O_2$ . However in the presence of strong detachment the current growth is determined by a more elaborate equation<sup>20</sup> (Dutton, 1978).



(a)



(b)

Fig. 8.6 Data on (a) the attachment coefficient  $\eta/N$  as a function of  $E/N$ , (b) the ionization coefficient. Fitted curve is the dashed line. Experimental and theoretical data are indicated by letters 'e' and 't' in the legend. Numbers in the legend are references in the original paper (Raju and Liu, with permission of IEEE, 1995©)

**Table 8.7**

Investigations on  $\alpha$  AND  $\eta$  in SF<sub>6</sub> (Raju and Liu, 1995) E, Experimental; T, Theory.  
(©1995 IEEE)

| Authors               | E/N (Td)                 | E/T | Reference                                                                                        |
|-----------------------|--------------------------|-----|--------------------------------------------------------------------------------------------------|
| Bhalla and Craggs     | $270 \leq E/N \leq 480$  | E   | Proc. of Phys. Soc., <b>80</b> (1962) 151-160                                                    |
| Kline et. al.         | $285 \leq E/N \leq 2000$ | E/T | J. Appl. Phys., <b>50</b> (1979) 6789-6796                                                       |
| Boyd and Crichton     | $400 \leq E/N \leq 700$  | E   | Proc. IEE, <b>118</b> (1971) 1872-1877                                                           |
| Geballe and Harrison  | $250 \leq E/N \leq 2000$ | E   | Basic Processes of Gaseous discharges, L. B. Loeb, University of California Press, 1965, 375-476 |
| Maller and Naidu      | $350 \leq E/N \leq 4000$ | E   | IEEE Trans. Plasma Sci., <b>3</b> (1975) 205-208; Proc. IEE, <b>123</b> (1976) 107-108.          |
| Teich and Sangi       | $700 \leq E/N \leq 7000$ | E   | Proc. Symp. on H. V. Technology, Munich, (1972) 391-395                                          |
| Urquijo-Carmona       | $200 \leq E/N \leq 7000$ | E   | Ph. D. Thesis, Univ. of Manchester, England, (1980)                                              |
| Yoshizawa et. al.     | $46 \leq E/N \leq 607$   | T   | J. Phys. D., Appl. Phys., <b>12</b> (1979) 1839-1853                                             |
| Novak and Frechette   | $120 \leq E/N \leq 750$  | T   | J. Appl. Phys., <b>53</b> (1982) 8562-8567                                                       |
| Dincer & Govinda Raju | $300 \leq E/N \leq 570$  | T   | J. Appl. Phys., <b>54</b> (1983) 6311-6316                                                       |
| Govinda Raju & Dincer | $300 \leq E/N \leq 1350$ | T   | J. Appl. Phys., <b>53</b> (1982) 8562-8567                                                       |
| Itoh et. al.          | $225 \leq E/N \leq 600$  | T   | J. Phys. D: Appl. Phys., <b>13</b> (1979) 1201-1209                                              |
| Itoh et. al.          | $210 \leq E/N \leq 480$  | T   | J. Phys. D: Appl. Phys., <b>12</b> (1979) 2167-2172                                              |
| Itoh et. al.          | $141 \leq E/N \leq 707$  | T   | J. Phys. D: Appl. Phys., <b>21</b> (1988) 922-930                                                |
| Itoh et. al.          | $71 \leq E/N \leq 7656$  | T   | J. Phys. D: Appl. Phys., <b>23</b> (1990) 299-303                                                |
| Itoh et. al.          | $141 \leq E/N \leq 707$  | T   | J. Phys. D: Appl. Phys., <b>23</b> (1990) 415-421                                                |
| Yousfi et. al.        | $100 \leq E/N \leq 960$  | T   | J. Phys. D: Appl. Phys., <b>18</b> (1985) 359-375                                                |
| Siddagangappa et. al. | $270 \leq E/N \leq 720$  | E   | J. Phys. D: Appl. Phys., <b>15</b> (1983) 763-772                                                |
| Aschwanden            | $150 \leq E/N \leq 918$  | E   | Fourth Inter. Conf. on Gaseous Electronics, Knoxville, Pergamon Press, (1994) 23-33              |
| Edelson and McAfee    | $50 \leq E/N \leq 200$   | E   | Rev. Sci. Instr. <b>35</b> (1964) 187-190                                                        |
| Raju et. al.          | $200 \leq E/N \leq 700$  | T   | J. Appl. Phys., <b>52</b> (1981) 3912-3920                                                       |
| Liu & Govinda Raju    | $100 \leq E/N \leq 600$  | E   | IEEE Trans. Plas. Sci., <b>20</b> (1992) 515-514                                                 |

The coefficients  $\alpha$ ,  $\eta$ ,  $\delta$  and  $\gamma$  are dependent on the applied electric field and the gas number density. It has been found experimentally that the first three coefficients expressed in a normalized way, that is  $\alpha/N$ ,  $\eta/N$  and  $\delta/N$ , are dependent on the ratio of E/N only and not on the individual values of E and N. The relationship between  $\alpha/N$  and E/N is expressed by the Townsend's semi-empirical relationship

$$\frac{\alpha}{N} = F \exp\left(-\frac{GN}{E}\right) = f(E/N) \quad (8.11)$$



where  $F$  and  $G$  are characteristics of the gas. It can be shown that the ratio  $G/F$  is approximately equal to the ionization potential of the gas. Table 8.8 shows the values of these parameters for several gases.

According to equation (8.6) the current in the gap is a maximum for the maximum value of  $\alpha$  at constant  $d$ . Differentiating equation (8.11) with respect to  $N$  to find the point of maximum  $\alpha$  yields

$$\frac{d\alpha}{dN} = \frac{d}{dN} \left[ N f\left(\frac{E}{N}\right) \right] = 0 \quad (8.12)$$

Differentiating the middle term yields

$$f\left(\frac{E}{N}\right) - \frac{E}{N} f'\left(\frac{E}{N}\right) = \frac{\alpha}{N} - \frac{E}{N} f'\left(\frac{E}{N}\right) = 0 \quad (8.13)$$

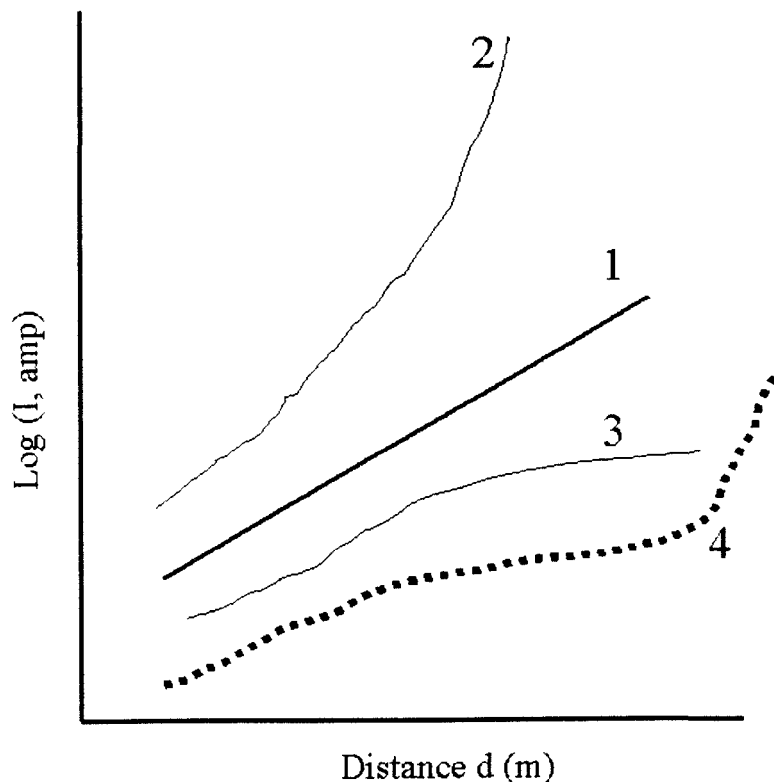


Fig. 8.7 Schematic current growth curves in a gap at various  $E/N$ . Curves 1 and 2 are observed in a non-attaching gas. Curves 3 & 4 are observed in an electron attaching gas.

This equation is rearranged to yield the condition for the maximum value of  $\alpha$  as a function of  $N$ ,

$$\frac{\alpha/N}{E/N} = f' \left( \frac{E}{N} \right) = \tan \theta \quad (8.14)$$

Equation (8.14) predicts that the current will be a maximum at the point of intersection of the tangent from the origin to the  $\alpha/N$  versus  $E/N$  curve (Fig. 8.8). This point is called the **Stoletow point**. At this point the value of  $E/N$  is equal to  $G$  in equation (8.11). Table 8.9 lists the values of  $(E/N)_{\text{Sto}}$  and  $(\alpha/N)_{\text{Sto}}$  at the Stoletow point for several gases in addition to the  $G/F$  and the ionization potential. The co-efficient  $\eta$ , defined as the ionization per volt ( $\alpha/E$ ), is sometimes found in the literature in place of  $\alpha$ . The expression for maximum  $\alpha$  may be obtained as follows. Differentiation of equation (8.11) yields

$$\frac{d\alpha}{dN} = F \left[ 1 - \frac{NG}{E} \right] \exp \left( -\frac{GN}{E} \right) = 0 \quad (8.15)$$

**Table 8.8**  
Gas constants in expression (8.11)

| Gas              | $F \times 10^{22} \text{ (m}^2\text{)}$ | $G \times 10^{22} \text{ (V m}^2\text{)}$ |
|------------------|-----------------------------------------|-------------------------------------------|
| Argon            | 372.7                                   | 6211                                      |
| Air              | 378.9                                   | 11335                                     |
| CO <sub>2</sub>  | 621.1                                   | 14472                                     |
| H <sub>2</sub>   | 329.2                                   | 10869                                     |
| HCl              | 776.4                                   | 11801                                     |
| He               | 56.5                                    | 1553                                      |
| Hg               | 621.1                                   | 11491                                     |
| H <sub>2</sub> O | 400.6                                   | 8975                                      |
| Kr               | 450.3                                   | 6832                                      |
| N <sub>2</sub>   | 329.2                                   | 10621                                     |
| Ne               | 124.2                                   | 3105                                      |
| Xe               | 689.4                                   | 9627                                      |

Equating the term in square brackets to zero yields the condition for  $N$  at which the current is a maximum as

$$N_{\max} = \frac{E}{G} \quad (8.16)$$

The Stoletow point corresponds to the minimum of the Paschen breakdown curve for gases, discussed in the next section. The ratio  $G/F$  is known as the effective ionization potential and in every case it is greater than the actual ionization potential due to the fact that the latter is obtained from beam experiments in which an electron beam of the specific energy passes through the gas. The physical significance of the Stoletow point is that at this value of  $E/N$  the energy for generating an ion-pair is a minimum resulting in a minimum sparking potential of the gas.

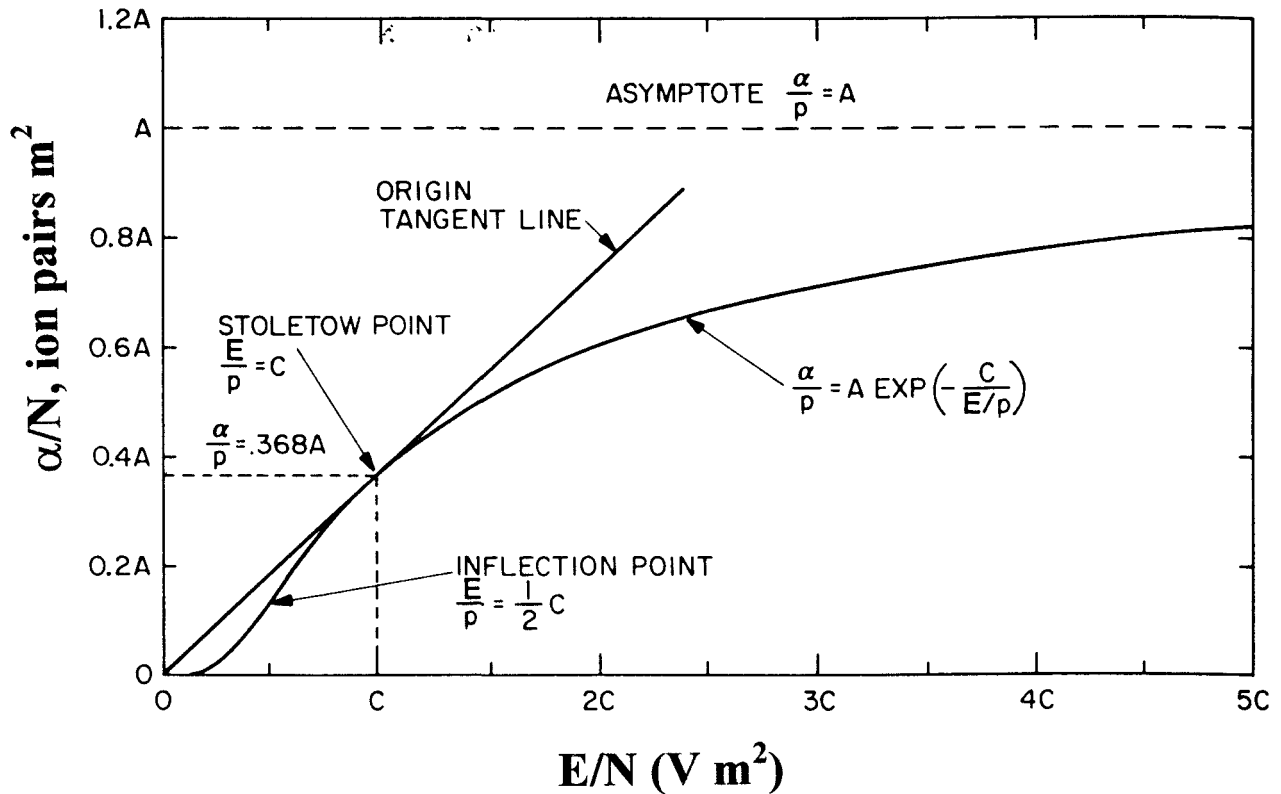


Fig. 8.8 Townsend's first ionization co-efficients plotted against the reduced electric field, (8.11) with major features of the curve indicated (Roth, 1995, with permission of Institute of Physics, UK.)

### **8.3 CRITERIA FOR BREAKDOWN**

Electron multiplication at low gas pressures under sufficiently high electric fields results in a regeneration of secondary electrons, each primary electron resulting in a secondary

electron. This condition is known as electrical breakdown of the gas and the discharge is said to be self sustaining. The current due to the electrons will be maintained even though the external agency, which provides the initiating electrons, is switched off.

Mathematically the condition is expressed by equating to zero the denominator of the growth equations (8.8) to (8.10).

### **A. Non-attaching gases**

$$\{\gamma(e^{\alpha d} - 1) - 1\} = 0 \quad (8.17)$$

Due to the large exponential term  $e^{\alpha d}$  the condition  $e^{\alpha d} \gg 1$  holds true and equation (8.17) simplifies to

$$\gamma e^{\alpha d} = 1 \quad (8.18)$$

This equation is known as the Townsend's criterion for breakdown.

### **B. Attaching gases**

The condition for breakdown in electron-attaching gases is given as

$$1 - \left\{ \frac{\gamma\alpha}{(\alpha - \eta)} [\exp(\alpha d) - 1] \right\} = 0 \quad (8.19)$$

Taking the natural logarithm of equation (8.19) we obtain

$$\ln\left(1 + \frac{1}{\gamma}\right) = \alpha d \quad (8.20)$$

These breakdown conditions lead to Paschen's law.

## **8.4 PASCHEN'S LAW**

Paschen discovered experimentally that the sparking potential of a uniform field gap is dependent on the product  $Nd$  rather than, on individual values of  $N$  and  $d$ . According to equation (8.11)  $\alpha/N$  is a function of  $E/N$  which may be rewritten as  $\alpha/N = \phi(V_s/Nd_s)$

where the suffix  $s$  denotes quantities when sparking occurs. If  $\gamma$  is also a function of  $E/N$  then  $\gamma = F(V_s/Nd_s)$ . Substituting these values in equation (8.18) we may write

$$FNd_s \left[ \exp \left( \frac{-GNd_s}{V_s} \right) \right] = \ln \left( 1 + \frac{1}{\gamma} \right) \quad (8.21)$$

This expression leads to

$$V_s = \frac{GNd_s}{\ln \left[ \frac{FNd_s}{1 + 1/\gamma} \right]} \quad (8.22)$$

which may be rewritten as

**Table 8.9**  
Stoletow Constants for gases (Roth, 1995)

| Gas              | $(E/N)_{\text{Sto}}=G$<br>(V m <sup>2</sup> ) | $(\alpha/N)_{\text{Sto}}$<br>$\times 10^{22}$ m <sup>2</sup> | $\eta = E/\alpha$<br>V | G/F  | $\varepsilon_i$ (eV) |
|------------------|-----------------------------------------------|--------------------------------------------------------------|------------------------|------|----------------------|
| Argon            | 6211                                          | 137.6                                                        | 45                     | 16.7 | 15.7                 |
| Air              | 11335                                         | 139.4                                                        | 81                     | 29.9 | -                    |
| CO <sub>2</sub>  | 14472                                         | 228                                                          | 63                     | 23.3 | 13.7                 |
| H <sub>2</sub>   | 10869                                         | 121                                                          | 90                     | 33.0 | 15.4                 |
| HCl              | 11801                                         | 286                                                          | 41                     | 15.2 | -                    |
| He               | 1553                                          | 21                                                           | 74                     | 27.5 | 24.5                 |
| Hg               | 11491                                         | 228                                                          | 50                     | 18.5 | 10.4                 |
| H <sub>2</sub> O | 8975                                          | 147                                                          | 61                     | 22.4 | 12.6                 |
| Kr               | 6832                                          | 165                                                          | 41                     | 15.2 | 14                   |
| N <sub>2</sub>   | 10621                                         | 121                                                          | 88                     | 32.2 | 15.5                 |
| Ne               | 3105                                          | 46                                                           | 68                     | 25.0 | 21.5                 |
| Xe               | 9627                                          | 254                                                          | 38                     | 14.0 | 12.1                 |

(with permission from Institute of Physics, Bristol)

$$V_s = f(Nd_s) \quad (8.23)$$

The nature of the function  $f$  must be determined experimentally for each gas.

Each gas has a minimum sparking voltage, called the Paschen minimum, below which breakdown cannot occur. To obtain an expression for the minimum voltage we use equation (8.22). By differentiating this expression with respect to the product  $Nd$  and equating the derivative to zero the value of  $Nd_{min}$  can be shown to be

$$Nd_{min} = \frac{e}{F} \ln(1 + \frac{1}{\gamma}) = \frac{2.718}{F} \ln(1 + \frac{1}{\gamma}) \quad (8.24)$$

where the parameter  $F$  is already listed in Table 8.8 for several gases. A dimensionless reduced  $Nd$  may be defined according to

$$x \equiv (Nd)_{red} = \frac{Nd}{(Nd)_{min}} \quad (8.25)$$

A dimensionless reduced sparking voltage may also be defined according to

$$y \equiv \frac{V_s}{V_{s,min}} \quad (8.26)$$

Substituting equations (8.25) and (8.26) in (8.22) yields

$$y \equiv \frac{V_s}{V_{s,min}} = \frac{x}{1 + \ln x} \quad (8.27)$$

A plot of  $y$  versus  $x$  gives a universal Paschen curve and it has the following features. At low values of  $x \ll 1$  the curve rises asymptotically at the left reaching a value of infinity at  $x = -1$ . The co-ordinates of the lowest point on the curve are of course (1,1). On the right hand side the curve rises somewhat linearly on the right due to the logarithmic term in the denominator.

The breakdown voltage on the left of the minimum increases because the mean free path for ionizing collisions increases and the number of ionizing collisions is correspondingly reduced. The number of ionizing collisions is also small because of the reduced number of neutral gas molecules for ionization. On the other hand, at higher gas pressures, on the right hand side of the minimum, the number of neutral molecules is so great that frequent collisions occur. Such increased frequency of collisions does not allow the electron energy to build up to the ionization limit, thus requiring higher voltage for breakdown. Theoretically, breakdown is impossible at  $\ln x = -1$  in equation (8.27), but

this is not true in practice because other mechanisms such as field emission come into play at such low values of  $Nd$ .

## **8.5 BREAKDOWN TIME LAGS**

Electrical breakdown of a gap does not occur instantly although the applied electrical field has the critical magnitude to satisfy the criterion, equation (8.20). The time interval from the instant of application of voltage to complete breakdown is called the time lag and arises out of two reasons:

1. The time required for one or more initial electrons to appear in a favourable position in the gap to lead to the necessary avalanche. This is called the statistical time lag.
2. The time required for these electrons to build up the primary avalanche and succeeding generations that leads to a current rise at breakdown. This is called the formative time lag.

### **8.5.1 THE STATISTICAL TIME LAG**

The statistical time lag depends upon the nature and degree of irradiation, cathode material and surface conditions such as the presence of oxidized layer. If  $\beta$  is the rate at which electrons are produced in the gap by external irradiation,  $P_1$  the probability of an electron appearing in a region of the gap where it can lead to a breakdown and  $P_2$  the probability that it will actually lead to a spark, the average statistical time lag is given by

$$\tau_s = \frac{1}{\beta P_1 P_2} \quad (8.28)$$

If the gap has not broken down for a time interval  $t$  the probability that it will breakdown in the next interval of time  $dt$  is  $\beta P_1 P_2 dt$ .

Let us consider  $n_o$  number of voltage applications out of which  $n_t$  is the number of breakdowns with a time lag greater than  $t$ . In the next interval  $dt$  the number of breakdowns will be

$$dn = -\beta P_1 P_2 n dt \quad (8.29)$$

The total number of breakdowns is obtained by integrating this expression as

$$\int_{n_0}^{n_t} \frac{dn}{n} = - \int_0^t \beta P_1 P_2 dt \quad (8.30)$$

Assuming that  $P_1$  and  $P_2$  are independent of  $t$  we get

$$\log \left( \frac{n_t}{n_0} \right) = -\beta P_1 P_2 t \quad (8.31)$$

which yields

$$n_t = n_0 \exp \left( -\frac{t}{\tau_s} \right) \quad (8.32)$$

$\tau_s$  is called the mean statistical time lag and equation (8.32) is known as the Laue equation. A plot of  $\ln(n_t/n_0)$  yields a straight line having a slope of  $-1/\tau_s$  as shown in Fig. 8.9. The time lags decrease with an increasing over voltage and an increasing level of irradiation. The time lag corresponding to  $n_t/n_0 = 1$  gives the formative lag for the particular conditions.

### **8.5.2 FORMATIVE TIME LAGS IN UNIFORM FIELDS**

The formative time lag is determined mainly by the fundamental processes and depends upon the gas. Other parameters that influence the formative times are the electrode geometry and the magnitude of the over-voltage. Measured formative time lags can theoretically be interpreted in terms of the avalanche build up and this requires calculation of the electron multiplication as a function of time and space. Let us consider a uniform field gap at moderate pressures of 10-100 kPa. The primary and secondary ionization mechanisms are considered to be the growth components. The continuity equations for the electrons and positive ions may be expressed as

$$\frac{\partial n_e(x,t)}{\partial t} = W_e \alpha n_e(x,t) - \frac{W_e \partial n_e(x,t)}{\partial x} \quad (8.33)$$

$$\frac{\partial n_p(x,t)}{\partial t} = W_p \alpha n_p(x,t) + \frac{W_p \partial n_p(x,t)}{\partial x} \quad (8.34)$$



Here  $n_e$  and  $n_p$  are the number of electrons and positive ions, respectively;  $\alpha$  the Townsend's first ionization co-efficient;  $W_e$  and  $W_p$  the drift velocity of electrons and positive ions, respectively,  $x$  the spatial variable and  $t$  the temporal variable. The left hand terms in equations (8.33) and (8.34) denote the temporal rate at which the electrons and the positive ions increase. The first term on the right hand side is the increase due to ionization by collision and the second term is due to the electrons (positive ions) leaving (arriving) the element of distance  $dx$ . Note the negative sign for the electrons leaving element  $dx$ .

The analytical solution of these two equations is obtained by Davidson<sup>21</sup> by applying the initial and boundary conditions:

$$\begin{aligned}
 \text{i.} \quad & n_e(x, t) = 0 & t \leq 0 \\
 \text{ii.} \quad & n_p(x, t) = 0 & t \leq 0 \\
 \text{iii.} \quad & n_p(d, t) = 0 & t > 0 \\
 \text{iv.} \quad & n_e(0, t) = n_0 + \gamma n_p(0, t) + \delta \int_0^d n_e(x, t) e^{-\mu x} & (8.35)
 \end{aligned}$$

where  $n_0$  is the number of initial electrons,  $\gamma$  the secondary ionization co-efficient due to positive ion action at the cathode, and  $\delta$  the photo-electric efficiency (number of secondary electrons generated per primary electron). Initial conditions (i) and (ii) apply when there are no charge carriers in the gap before the application of the voltage, boundary condition (iii) applies because at the anode there ( $x = d$ ) are no positive ions. Boundary condition (iv) shows that both secondary mechanisms (positive ions and photons) are taken into account.

The number of electrons at  $(x, t)$  is given by

$$n_e(x, t) = n_e(0, t - \frac{x}{W_e}) e^{\alpha x} \quad (8.36)$$

Application of Laplace transforms to equations (8.33) and (8.34) gives

$$\frac{s}{W_e} n_e(x) = \alpha n_e(x) - \frac{\partial n_e(x)}{\partial x} \quad (8.37)$$

$$\frac{s}{W_p} n_p(x) = \alpha n_e(x) + \frac{\partial n_p(x)}{\partial x} \quad (8.38)$$

Applying the transformed boundary conditions yields the solution of these equations,

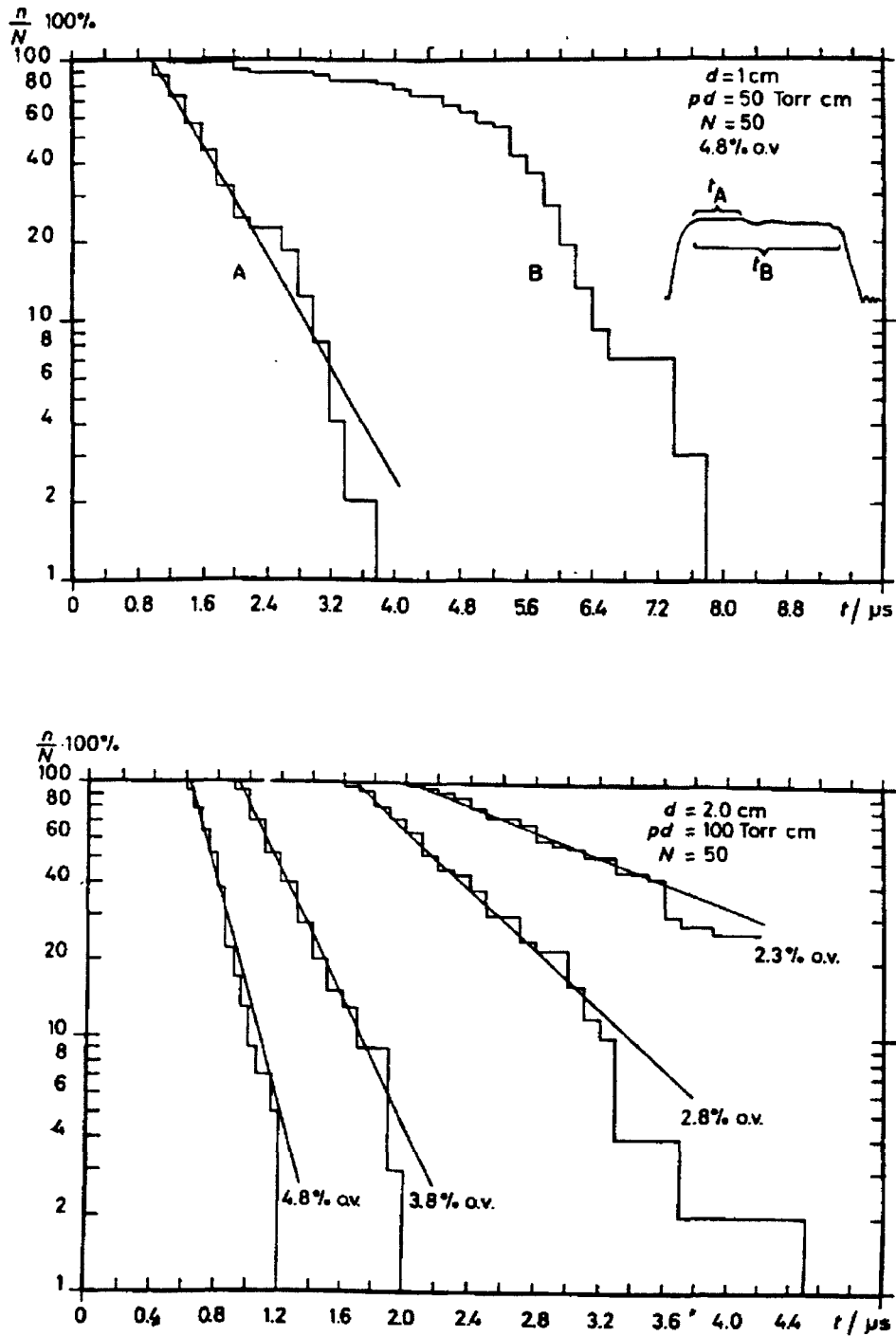


Fig. 8.9 Linear and non-linear time lag distributions in  $SF_6$  (Meek and Craggs, 1978, with permission of John Wiley and Sons, New York).

$$n_e(x) = Ae^{\psi x} \quad (8.39)$$

$$n_p(x) = \frac{A\alpha}{\phi} e^{\psi x} + Be^{sx/w_p} \quad (8.40)$$

where  $\phi = \alpha - s/W_e$ ,  $\psi = \alpha - \mu - (s/W_p)$  and A and B are constants given by

$$A = \frac{n_o}{s \left[ 1 - (\alpha\gamma/\phi)(e^{\phi d} - 1) + (\delta/\psi)(e^{\psi d} - 1) \right]} = \frac{n_o}{s f(s)} \quad (8.41)$$

and

$$B = \frac{\alpha A e^{\phi d}}{\phi} \quad (8.42)$$

where  $n_o$  is the initial number of electrons released from the cathode. Substituting equations (8.41) and (8.42) in (8.39) and (8.40) we get the solution for the number of electrons and positive ions as

$$n_e(x) = n_o \frac{e^{\psi x}}{s f(s)} \quad (8.43)$$

$$n_p(x) = \frac{\alpha n_o e^{sx/w_p}}{s f(s)} \left\{ \frac{e^{\phi d} - e^{\phi x}}{\phi} \right\} \quad (8.44)$$

The solution is obtained by finding the inverse Laplace transforms of equations (8.43) and (8.44). The details of computation are explained in the following section.

### **8.5.3 Formative Time Lags in Cylindrical Geometry**

We consider a cylindrical geometry<sup>22</sup> with the inner and outer electrodes having a radii of  $R_1$  and  $R_2$ , respectively because this geometry is more relevant to practical applications. Let  $r$  be the spatial variable, and as in the previous case, we consider a source of external irradiation which liberates uniformly a constant initial current of  $i_o$

from the cathode and at  $t = 0$  a voltage is applied to the gap. The current increases temporally and spatially according to

$$\frac{1}{W_e} \frac{\partial i_-(r,t)}{\partial t} = \alpha(r) i_-(r,t) - \frac{\partial i_-(r,t)}{\partial r} \quad (8.45)$$

$$\frac{1}{W_p} \frac{\partial i_+(r,t)}{\partial t} = \alpha(r) i_+(r,t) + \frac{\partial i_+(r,t)}{\partial r} \quad (8.46)$$

where  $i_-$  and  $i_+$  are currents due to electrons and positive ions respectively. They replace  $n_e$  and  $n_p$  respectively. The initial and boundary conditions are:

$$\begin{aligned} \text{i. } & i_-(r,t) = 0, \quad t \leq 0 \\ \text{ii. } & i_+(r,t) = 0, \quad t \leq 0 \\ \text{iii. } & i_+(R_2,t) = 0 \\ \text{iv. } & i_-(R_1,t) = i_0 + \gamma_+ i_+(R_2,t) + \gamma_{ph} \int_{R_2}^{R_1} \alpha(r) i_-(r,t) dr \end{aligned} \quad (8.47)$$

where  $\gamma_+$  and  $\gamma_{ph}$  are the relative contributions of positive ions and photons, respectively, to the total secondary ionization co-efficient,  $\gamma_T$ . Hence

$$\gamma_T = \gamma_+ + \gamma_{ph} \quad (8.48)$$

The Townsend's first ionization co-efficient is a function of  $E/N$  according to equation (8.11) and, since the field varies radially according to

$$E(r) = \frac{V}{r \ln(R_2 / R_1)} \quad (8.49)$$

the radial variation of  $\alpha$  is taken into account according to

$$\alpha(r) = FN \exp \left[ -\frac{GN}{E(r)} \right] \quad (8.50)$$

Substituting equation (8.49) in (8.50) we get

$$\alpha(r) = FN \exp(-cr) \quad (8.51)$$

where

$$c = \frac{GN}{V} \ln(R_2 / R_1) \quad (8.52)$$

If we wish to use equation (8.47) as a boundary condition the solutions of equations (8.45) and (8.46) become extremely complicated and therefore we make an approximation that at voltages below breakdown and at the threshold voltage at least 90% of the positive ions are generated near the anode. This yields the equation

$$i_{-(R_1, t)} = i_0 + i_{+(R_2, t-t_i)} A \quad (8.53)$$

in which  $t_i$  is the ion transit time and A is called the regenerative factor, defined by

$$A = \gamma_T \left\{ \left[ \exp \int_{R_1}^{R_2} \alpha(r) dr \right] - 1 \right\} \quad (8.54)$$

The ion transit time is given by

$$t_i = \left( \frac{R_2 - R_1}{W_p} \right) \quad (8.55)$$

For  $t \gg t_i$  we can make the approximation

$$i_{+(R_2, t-t_i)} = i_{+(R_2, t)} - t_i \frac{d[i_{+(R_2, t)}]}{dt} \quad (8.56)$$

Substituting equation (8.56) in (8.53) and omitting the suffixes for the sake of simplicity we obtain the differential equation

$$i = i_0 + A[i - t_i i'] \quad (8.57)$$

the solution of which is given by

$$i = \frac{i_o}{A-1} \left\{ \exp \left[ \left( \frac{A-1}{A} \right) \frac{t}{t_i} \right] - 1 \right\} \quad (8.58)$$

If we use equation (8.58) as the boundary condition in place of (8.47) we can obtain the analytical solutions of (8.45) and (8.46).

Taking Laplace transforms the currents are expressed as

$$I_{-(r,s)} = K_1 e^{\frac{-sr}{W_e} - \frac{FN}{c} \exp(-cr)} \quad (8.59)$$

$$I_{+(r,s)} = K_3 e^{\frac{sr}{W_p}} + \frac{FNK_2 e^{\frac{sR_1}{W_e}} e^{-r(c + \frac{s}{W_e})W_{eff}}}{At_1 s \left[ s + \frac{(1-A)}{At_i} \right] \left[ s + cW_{eff} \right]} \quad (8.60)$$

in which  $K_1$  and  $K_2$  are constants to be determined from the transformed boundary conditions, and

$$K_2 = i_0 e^{\frac{FN}{c}} [\exp(-cR_1) - (\exp(-cR_2))] \quad (8.61)$$

In equations (8.60) and (8.61) we have defined the following quantities

$$\frac{1}{W_{eff}} = \frac{1}{W_e} + \frac{1}{W_p} \quad (8.62)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (8.63)$$

Hence

$$K_1 = \frac{i_0 e^{\frac{sR_1}{W_e} + \frac{FN}{c} \exp(-cR_1)}}{s[1 + At_1 s - A]} \quad (8.64)$$

$$K_3 = - \frac{FNK_2 W_{eff} e^{\frac{sR_1}{W_e}} e^{-(c+\frac{s}{W_e})(R_2-R_1)-\frac{s}{W_p}(R_2-R_1)}}{At_i s \left[ s + \frac{(1-A)}{At_i} \right] \left[ s + cW_{eff} \right]} \quad (8.65)$$

Substituting equations (8.61), (8.64) and (8.65) in equations (8.59) and (8.60) and taking inverse Laplace transforms yields

$$\begin{aligned} i_{-(r,t)} &= \frac{K_2}{(A-1)} \left\{ \exp \left[ \left( \frac{A-1}{At_i} \right) \left( t + \frac{R_1}{W_e} - \frac{r}{W_e} \right) \right] - 1 \right\}; t \geq \left( \frac{r}{W_e} - \frac{R_1}{W_e} \right) \\ &= 0 : t \leq \left( \frac{r}{W_e} - \frac{R_1}{W_e} \right) \end{aligned} \quad (8.66)$$

$$\begin{aligned} i_{+(r,t)} &= FNK_2 \exp(-cR_2) \left\{ \frac{1}{c(A-1)} + \frac{At_i W_{eff}}{(A-1)(1-A-At_i cW_{eff})} e^{\left( \frac{A-1}{At_i} \right) \left\{ t - \left[ \frac{(R_2-R_1)}{W_e} + \frac{R_2}{W_i} - \frac{r}{W_i} \right] \right\}} \right. \\ &\quad \left. - \frac{1}{c(A-1+cW_{eff}At_i)} e^{-cW_{eff} \left[ t - \frac{(R_2-R_1)}{W_e} + \frac{(R_2-r)}{W_i} \right]} \right\} \\ &\quad - FNK_2 \exp(-cr) \left\{ \frac{1}{c(1-A)} + \frac{AW_{eff}t_i}{(At_i cW_{eff} + A-1)(A-1)} e^{\left( \frac{A-1}{At_i} \right) \left[ t - \left( \frac{r}{W_e} - \frac{R_1}{W_e} \right) \right]} \right. \\ &\quad \left. - \frac{1}{c(1-A-AcW_{eff}t_i)} \exp \left[ -cW_{eff} \left( t - \frac{r}{W_e} + \frac{R_1}{W_e} \right) \right] \right\} \end{aligned} \quad (8.67)$$

If we substitute

$$i_{+(r,t)} = P_1 + P_2$$

then

$$\left. \begin{aligned} P_1 &= 0; t < \frac{(R_2 - R_1)}{W_e} + \frac{R_2}{W_i} - \frac{r}{W_i} \\ P_2 &= 0; t < \frac{r}{W_e} - \frac{R_1}{W_e} \end{aligned} \right\} \quad (8.68)$$

Equation (8.67) gives the positive ion current at  $(r, t)$  subject to the condition imposed by equation (8.68). The total current at  $(r, t)$  is given by

$$i_{(r,t)} = i_{-(r,t)} + i_{+(r,t)} \quad (8.69)$$

The current from the cathode at time  $t$  is given by

$$i_{-(R_1,t)} = i_0 + \gamma_+ i_{+(R_1,t)} + \gamma_{ph} i_{-(R_2,t)} \quad (8.70)$$

This is the true value of  $i_0$  which should be used in the calculation of current multiplication which is given by

$$M = \frac{i_{-(R_2,t)}}{i_0} = \frac{i_{-(R_1,t)}}{i_0(A-1)} \left\{ \exp \left[ \frac{(A-1) \left( t + \frac{R_1}{W_e} - \frac{r}{W_e} \right)}{A t_{eff}} \right] - 1 \right\} \quad (8.71)$$

in which the effective transit time is defined according to Heylen's<sup>23</sup> derivation

$$\frac{1}{t_{eff}} = \frac{\gamma_+}{\gamma_T} \frac{1}{t_i} + \frac{\gamma_{ph}}{\gamma_T} \frac{1}{t_e} \quad (8.72)$$

The formative time lag is given by that value of  $t$  in equation (8.71) for which a specified multiplication of current occurs for breakdown to become inevitable.

To demonstrate the applicability of the theory the following experimental conditions are employed:  $R_1 = 0.7$  cm,  $R_2 = 3.1$  cm,  $N = 3.54 \times 10^{21} \text{ m}^{-3}$  and  $1.75 \times 10^{21} \text{ m}^{-3}$ . Very good agreement between measured and calculated<sup>24</sup> time lags are obtained as shown in Fig. 8.10.



## **8.6 THE STREAMER MECHANISM**

The theory of electrical breakdown based on the regenerative mechanism of secondary electrons at the cathode, mainly due to the photoelectric action at the cathode, is applicable to low gas pressures. Experimental evidence suggests that at high pressures ( $\sim 1$  atm. and above) streamers develop in the gap due to photo-ionization in the gas and breakdown occurs at time lags in the range of 1 ns-10 $\mu$ s, depending upon the over-voltage, gas pressure, gap length, etc.

The origin of the streamer inception is the number of electrons at the head of the advancing primary avalanche. When the number reaches a value in the range of  $10^7 - 10^8$  the electric field due to the space charge is sufficiently strong to allow secondary electrons due to photo-ionization to develop into the avalanche head. The streamer theory explains many details observed in non-uniform field breakdown in long gaps and electrical corona. The dependence of the corona inception voltage and breakdown voltage on the polarity of the applied voltage may also be explained using the streamer criterion. The streamer mechanism was first proposed by Meek (Meek and Craggs (1978) and independently by Raether<sup>25</sup> and many details have been clarified in the literature since then.

The streamer mechanism of electrical breakdown is based on the following considerations. The primary avalanche is initiated by a single electron and as the avalanche moves close to the anode there is a distribution of positive ions which is most intense at the head of the avalanche. The positive ions produce a field distortion in both the radial and axial direction. The axial component will add to the applied field, whereas the radial component will allow the electrons generated due to photo-ionization to grow into it. The radial component of the field should be of the same order of magnitude as the applied field for the successful inception of a streamer.

Fig. 8.11 shows the development of a secondary avalanche into the head of the advancing streamer. The secondary electrons generated due to photo-ionization in the gas should be within the active region which is defined as the region that allows for electron multiplication. In electron attaching gases the boundary of the active region is determined by the criterion  $\alpha = \eta$ . In air at atmospheric pressure this criterion is satisfied at 26 kV/cm and in SF<sub>6</sub> at 91 kV/cm. In low electric fields the streamer may still advance because of its own field provided the number of ions is approximately  $10^8$  and the avalanche head has a radius of 30  $\mu$ m<sup>26</sup>. The field in the active region may be calculated as

$$E = E_a + \frac{N_s e}{4\pi\epsilon_0 (z_0 + R_s)^2} \quad (8.73)$$

where  $E_a$  is the applied field,  $N_s$  the number of charges in the avalanche head of radius  $R_s$  and  $Z_0$  the starting point of the secondary avalanche along the axis (Fig. 8.11).

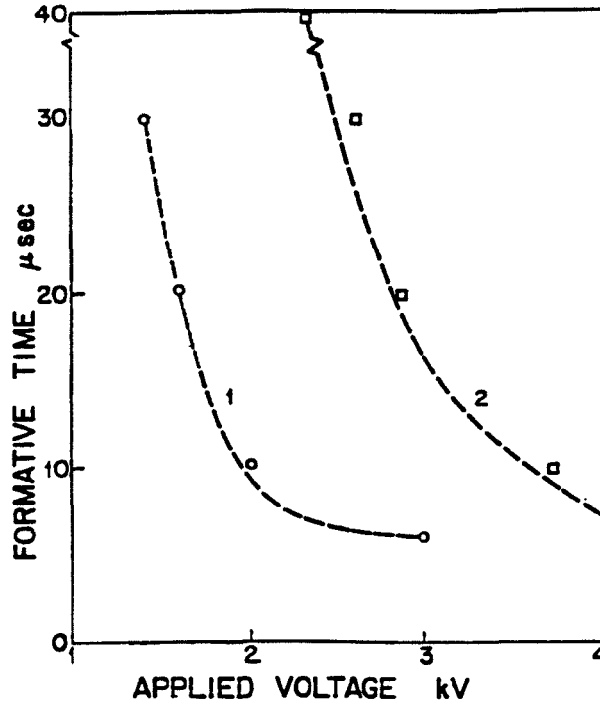


Fig. 8.10 Formative time lags in nitrogen at  $N = 3.54 \times 10^{21} \text{ m}^{-3}$  (curve 1) and  $N = 1.57 \times 10^{21} \text{ m}^{-3}$  (curve 2). Broken lines: theory, symbols: experimental. (Govinda Raju, 1983, with permission of American Institute of Physics).

Substituting typical values for  $\text{SF}_6$  91 kV/cm,  $N_s = 10^8$ ,  $z_0 = 80 \text{ } \mu\text{m}$ ,  $R_s = 30 \text{ } \mu\text{m}$  the field in the active region is  $E = 75 \text{ kV/cm}$  which is of the same order of magnitude as the applied voltage.

The secondary avalanche starts to grow in the active region as long as the energy gained by the drifting electrons from the applied field is balanced by the energy lost in collisions with gas molecules (ionization, attachment, momentum transfer etc.) and the energy stored as potential energy in the streamer tip. The energy balance equation is

$$\epsilon_e = \epsilon_a + \Delta \epsilon_{\text{pot}} \quad (8.74)$$

where  $\epsilon_c$  is the energy lost in collisions (momentum transfer, electronic excitation, dissociation, attachment, vibrational excitation and rotation, ionization), and  $\epsilon_{\text{pot}}$  the potential energy because of the concentration of the charges and their position in the applied field. Figure 8.12 shows the agreement between theoretical values based on the model and experimental results (Gallimberti, 1981).

The criterion for the initiation of the streamers from a single electron avalanche in high pressure electron attaching gases ( $p > 1$  bar) has been expressed as  $\alpha = \eta$ . This condition was initially derived by Harrison and Geballe (Loeb, 1965) and used by many authors to calculate or predict breakdown voltages. The physical significance of this criterion is that the electron avalanche ceases to grow at this limiting condition, often expressed as  $(E/N)_{\text{lim}}$  and therefore, strictly speaking, it does not lead to breakdown. The condition implies that if the voltage exceeds that given by  $(E/N)_{\text{lim}}$  by a small fraction ( $\sim 0.1\%$ ) the electron avalanche multiplies so rapidly that breakdown ensues. The limiting value is 86.4 V/m Pa for  $\text{SF}_6$ , 89.5 V/m Pa for  $\text{CCl}_2\text{F}_2$  and 23.6 V/m Pa for air.

### **8.6.1 Leader Mechanism**

The mechanisms of breakdown in long gaps, as in spark-over in a transmission line, discharges along bushings and in lightning are different from the streamer mechanism in uniform electric fields. Because of the very high voltages applied and the non-uniformity of the electric field, the field near the anode is very high. Intense ionization occurs close to the electrode and the electrons collide with neutral molecules transferring energy.

The local temperature may rise to as high as  $10^4$  K. Thermal ionization occurs and the area surrounding the electrode becomes a hot plasma channel, called the leader channel. The thermal ionization renders the channel highly conductive and the axial field is quite small. However the radial variation of the field is sharp. The head of the channel is approximately at the same potential as the electrode. The electric field ahead of the channel is intensified and several streamers are launched. The electrons from the new streamers flow back into the head of the channel and lengthen the leader until it extends to the cathode. The velocity of propagation of a leader can reach values as high as  $2 \times 10^6$  m/s.

Leader initiated breakdown may actually begin as corona at the high voltage electrode. Corona formation results in a pre-ionized region making possible the transition to a leader or facilitating the propagation of the leader. Figure (8.13) shows a beautiful

picture of corona formation in the gap while the leader has bridged the gap partially. The corona region adjacent to the leader is very similar to a glow discharge in characteristics. The region is almost charge neutral, with a low concentration of electrons and a high concentration of equal positive and negative ions. The resulting conductivity is quite low. Several models have been proposed for the transition of corona into a leader, one of them being the thermal ionization briefly referred to above.

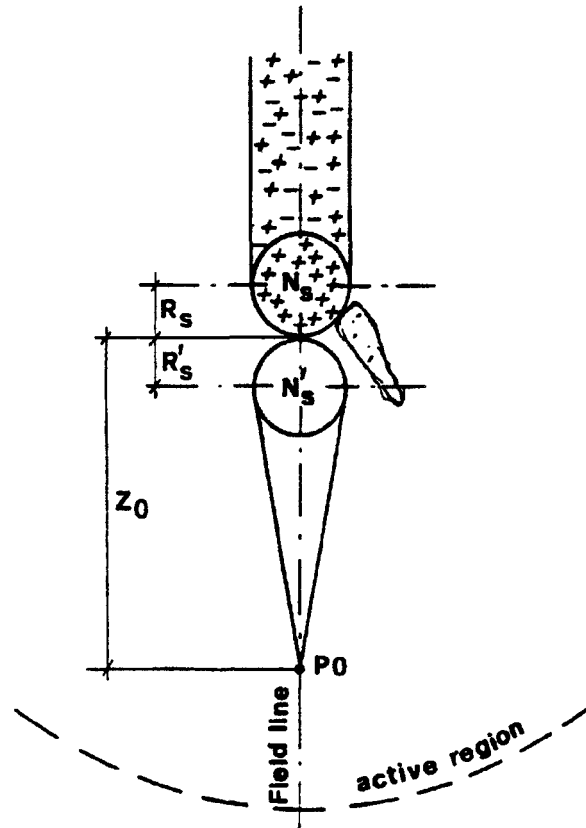


Fig. 8.11 Development of secondary avalanche into advancing streamer, adopted from (Gallimberti, 1981, with permission from Plenum Press, New York).

## **8.7 FIELD DISTORTION DUE TO SPACE CHARGE**

When the electron multiplication is large the difference in mobility of electrons and positive ions introduces a space charge which distorts the applied field. To calculate the inhomogeneity that arises due to space charge we consider a parallel plane gap with an applied voltage  $V$ , and the electric field is uniform initially across the gap length  $d$ . Multiplication of electrons occurs and the electric field at  $dx$  at a distance of  $x$  from the cathode is obtained by applying Poisson's equation,

$$\frac{dE}{dx} = -\frac{\rho}{\epsilon_0} \quad (8.75)$$

where  $\rho$  is the volume charge density. As a simplification we assume that the charge density is proportional to  $e^{\alpha x}$  where  $x$  is the distance over which the growth of avalanche has occurred.

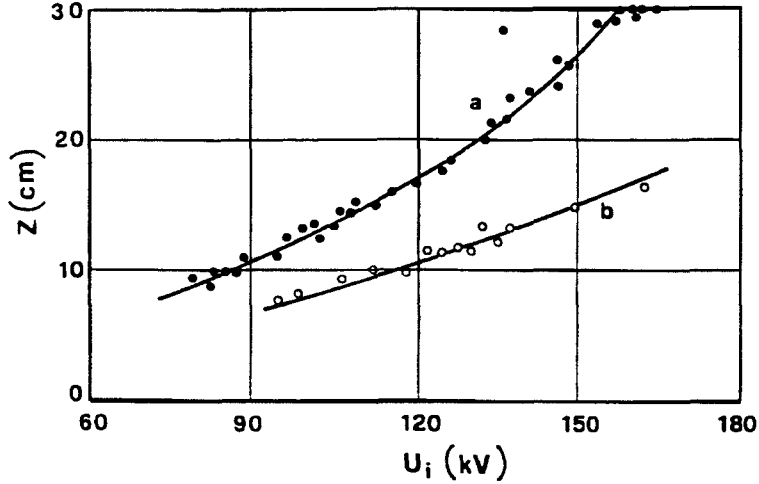


Fig. 8.12 The streamer length as a function of the inception voltage for gap lengths of 30 cm (curve a) and 150 cm (curve b); the solid lines represent the computed values; The points are experimental results (square cut rod of 1 cm radius). (Gallimberti, 1978, with permission of Plenum Press, New York.)

Substituting  $\rho = Ke^{\alpha x}$  where  $K$  is a constant having the dimension of  $C\ m^{-3}$  and integrating equation (8.75) we get the electric field at  $x$  as

$$E_x = -\frac{K}{\epsilon_0} \frac{e^{\alpha x}}{\alpha} + C \quad (8.76)$$

where  $C$  is the integration constant. At the cathode  $x = 0$  and  $E_x = E_c$ . Substituting these values in equation (8.76) we get

$$C = E_c + \frac{K}{\epsilon_0 \alpha} \quad (8.77)$$

Substituting equation (8.77) in (8.76) we get

$$E_x = \frac{K\rho}{\varepsilon_0 \alpha} - \frac{K\rho}{\varepsilon_0} \frac{e^{\alpha x}}{\alpha} + E_c \quad (8.78)$$

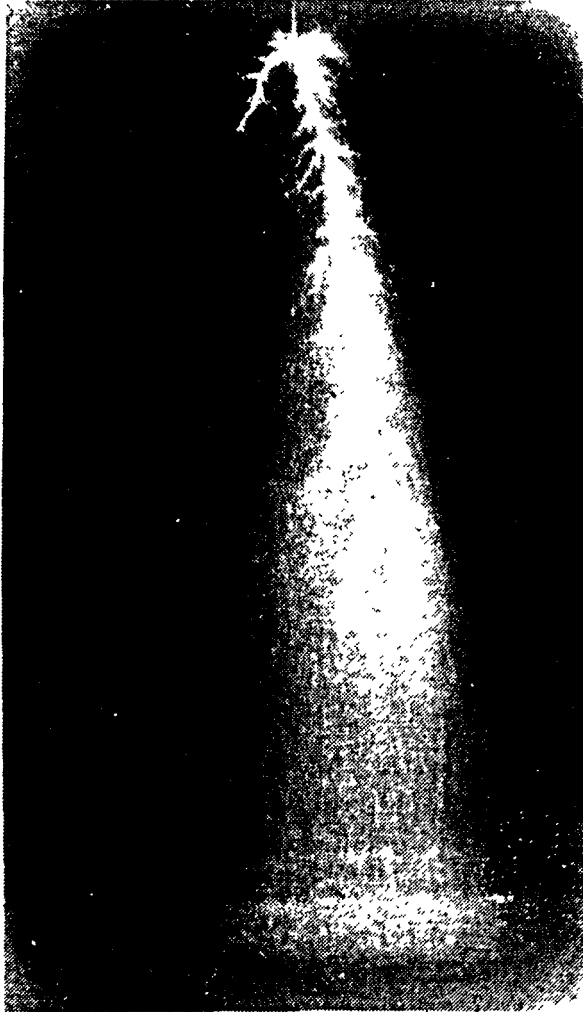


Fig. 8.13 Partially developed leader in 1.5 m gap. The leader ahead of the streamer can be clearly seen (Gallimberti, 1981, with permission of Plenum Press, New York).

The multiplication of electrons required to distort the applied electric field should be large, i.e.,  $e^{\alpha x} \gg 1$  and we can make the approximation

$$E_x = E_c - \frac{K\rho}{\varepsilon_0 \alpha} e^{\alpha x} \quad (8.79)$$

Since the applied voltage is constant we may express it as

$$\int_0^d E_x dx = V \quad (8.80)$$

Integration yields

$$-\frac{K\rho}{\varepsilon_0 x^2} [e^{\alpha x}]_0^d + E_c d = V \quad (8.81)$$

Substituting the limits and approximating  $(e^{\alpha d} - 1) \approx e^{\alpha d}$  we get

$$\frac{K\rho}{\varepsilon_0 x^2} [e^{\alpha d}] + E_c d = V \quad (8.82)$$

combining equations (8.82) and (8.79) we get

$$E_x = \frac{V}{d} + \frac{K\rho}{\varepsilon_0 d^2} \frac{e^{\alpha d}}{d} - \frac{K}{\varepsilon_0 \alpha} e^{\alpha x} \quad (8.83)$$

This equation may be rewritten as

$$E_x = E_0 + \frac{K\rho}{\varepsilon_0} \left( \frac{e^{\alpha d}}{\alpha d} - e^{\alpha x} \right) \quad (8.84)$$

where  $E_0$  is the applied field. The electric field at the cathode, i. e., at  $x = 0$  is

$$E_c = E_0 - \frac{K}{\varepsilon_0 \alpha} \frac{e^{\alpha d}}{\alpha d} \quad (8.85)$$

The field at the anode, i. e., at  $x = d$  is

$$E_a = E_0 - \frac{K}{\varepsilon_0 \alpha} e^{\alpha d} \quad \text{because } \alpha d \gg 1$$

We can calculate the current which increases the cathode field significantly, say by 10%. Let us assume that the applied voltage is 30 kV across 1 cm gap length in air at atmospheric pressure. The multiplication appropriate to these conditions is  $\alpha d \sim 18$ . Using equation (8.85) the constant  $K$  is evaluated as  $13.5 \times 10^{12} \text{ C/m}^3$ . The average diameter of the electron avalanche is approximately 0.1 mm and an approximate volume of  $5 \times 10^{-12} \text{ m}^3$ . Hence  $K = 1.6 \times 10^{-19} / (5 \times 10^{-12}) = 3 \times 10^{-6} \text{ C/m}^3$  and  $\rho = K e^{\alpha x} = 3 \times 10^{-6} \times e^{18 \times 0.01} = 3 \times 10^{-6} \text{ C/m}^3$ . The current density corresponding to 10% field distortion at the cathode is given by  $j = \rho \mu E_c$  where  $\mu$  is the mobility of positive ions ( $10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ ) and  $E_c = 1.1 \times 3 \times 10^6 \text{ V/m}$ . Substituting these values we obtain a current density of  $10 \text{ mA/m}^2$ . A current of this magnitude is observed experimentally at higher pressures emphasizing the need for taking space charge distortion into account.

## **8.8 SPARKOVER CHARACTERISTICS OF UNIFORM FIELD GAPS IN SF<sub>6</sub>**

A large volume of experimental data is available in the literature on the sparkover characteristics of uniform field gaps<sup>27</sup> in various gases and we will not attempt to present these data. However, data in SF<sub>6</sub> are plotted in figure (8.14) and compared with those in nitrogen. SF<sub>6</sub> has a higher breakdown voltage when compared with nitrogen and at atmospheric pressure the ratio of breakdown voltages is approximately 2.95. The higher dielectric strength is attributed to the electron attachment in SF<sub>6</sub>.

The ratio of  $V_s/Nd$  gives the value of  $E_s/N$  at which breakdown occurs and this quantity is also plotted in figure 8.9 for both gases. For a non-attaching gas such as N<sub>2</sub> the ratio  $E_s/N$  decreases at first rapidly but at larger values of  $Nd$  the decrease is rather more gradual. However for an electron attaching gas there is a limiting value of  $E_s/N$  below which breakdown does not occur. At this limiting value the condition  $\alpha/N = \eta/N$  holds and for SF<sub>6</sub> the limiting  $E/N = 361 \text{ (Td)}$ . The proof for the existence of a limiting value of  $E/N$  for electron attaching gases may be obtained as follows:

For electron-attaching gases the breakdown criterion is given by equation (8.19). Rearranging terms we get

$$\gamma = \frac{(\alpha - \eta)}{\alpha} \frac{1}{\exp[(\alpha - \eta)d] - 1} \quad (8.86)$$

Since  $\alpha/N$  and  $\eta/N$  are both functions of  $E/N$  the breakdown voltage is dependent on  $\alpha$  as well as  $(\alpha - \eta)$ . As the product  $Nd$  increases the reduced field  $E_s/N$  decreases as demonstrated in figure 8.14, thereby decreasing  $\alpha/N$  and increasing  $\eta/N$ .



As  $\alpha \rightarrow \eta$  at constant N equation (8.86) becomes

$$\gamma = \frac{Lt}{(\alpha - \eta) \rightarrow 0} \frac{(\alpha - \eta)}{\alpha [\exp(\alpha - \eta)d - 1]} \quad (8.87)$$

$$= \frac{1}{\alpha d} \quad (8.88)$$

When  $\alpha = \eta$  breakdown occurs for a value of  $\gamma$  given by equation (8.88).

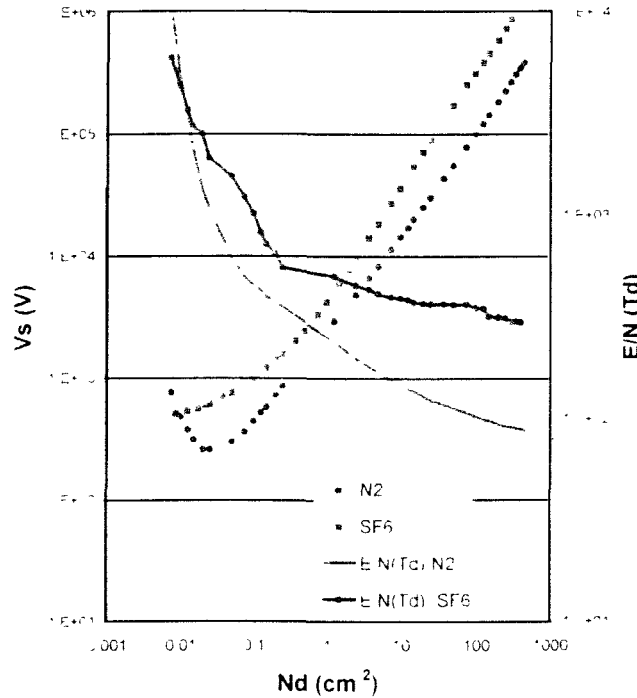


Fig. 8.14 Breakdown voltages in  $N_2$  and  $SF_6$  [data from Meek and Craggs, 1978]. (With permission of John Wiley and Sons, New York.)

If  $\alpha \ll \eta$  equation (8.86) becomes

$$\gamma = \frac{\eta}{\alpha} \frac{1}{(1 - \exp - \eta d)} \quad (8.89)$$

Both the terms on the right side are greater than unity and therefore breakdown occurs only if  $\gamma > 1$ . In electron attaching gases the secondary ionization co-efficient is usually

quite small, for example in  $\text{SF}_6$  it is of the order of  $10^{-5}$  and in oxygen  $\sim 10^{-4}$ . Therefore breakdown does not occur for  $\alpha < \eta$ .

## **8.9 SPARKOVER CHARACTERISTICS OF LONG GAPS**

Due to its engineering importance corona inception and breakdown of long gaps  $\geq 1\text{m}$  have been studied extensively in various electrode configurations, under d.c. and impulse voltages of various wave shapes. There is generally good understanding of the various features of discharges observed and a general picture may be drawn to characterize the influence of the gap length and the polarity on the corona inception ( $V_c$ ) and breakdown voltages ( $V_s$ ). Fig. (8.15) represents schematically the general characteristics in a point plane gap and sphere-plane gaps which present non-uniform electric fields of varying degree, depending upon the diameter of spheres<sup>28</sup>.

At small gap lengths on the order of a cm or less, it is difficult to detect corona before the sparkover takes place, and the curve for corona inception and sparkover coincide irrespective of the polarity of the point electrode. As the gap length is increased the corona inception voltage may be distinguished from the sparkover voltage and the difference between the two voltages increases as the gap length increases, making it easier to observe the polarity dependence of  $V_c$ . The gap length at which the onset of corona is observed is called the critical gap length.

The corona inception voltage for the negative polarity voltage is lower than that due to the positive polarity for all electrode geometries, possibly due to the fact that the electric field at the tip of the point is several times the average field in the gap. Initiating electrons are therefore more successfully launched into the gap as the avalanche grows from the high field electrode. Availability of a fortuitous electron to launch an avalanche, due to dissociation of a negative ions close to the negative point electrode, is also a contributing factor. The corona inception voltage for both positive and negative point electrode remains essentially the same as the gap length is increased. This is due to the fact that the gap length has relatively minor influence in lowering the electric field at the tip of the point electrode.

The sparkover voltage for positive polarity (curve 1, 3, 5 & 7 in Fig. 8.15) increases with the gap length and the difference between  $V_c$  and  $V_s$  increases as well. The initial phase of discharge in this region is an intense corona discharge from the point which develops into a leader. The positive leader is associated with a more diffuse discharge (Fig. 8.13) at its tip and is termed “leader-corona”.

After the passage of the leader tip the luminosity falls, until complete bridging of the gap causes the narrow channel to be reilluminated by the main discharge current ( Meek & Craggs, 1978). The return stroke which is sustained by the main current does not illuminate the branches. If the earthed electrode is a point or a rod then a negative leader propagates towards the positive leader. The junction of the two leaders occurs with a small region surrounding the junction point being brighter than the rest of the gap. The junction of the two leaders launches the return stroke. The negative leader velocity is about 30% of the positive leader.

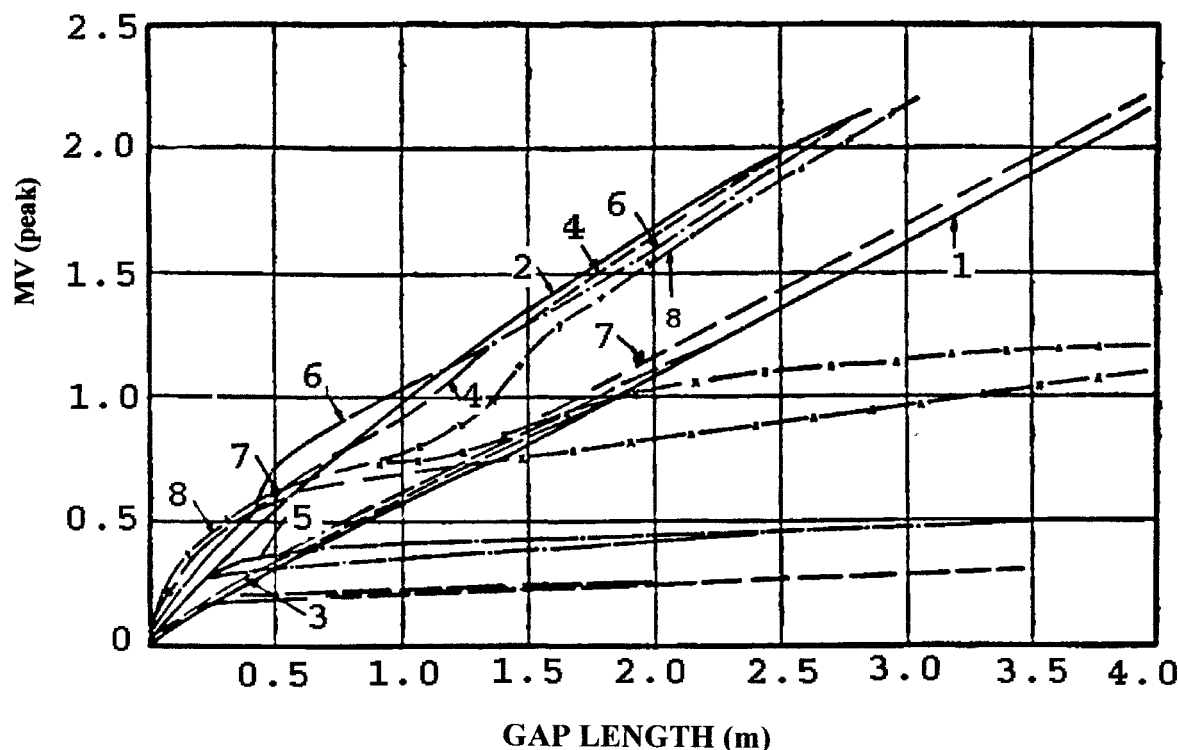


Fig. 8.15 Impulse breakdown voltage characteristics of large air gaps [Author's measurements]. The electrode geometry is: Breakdown voltage: 1&2, point-plane gap, positive and negative polarity, 3&4, 6.25 cm diameter sphere-plane gap, positive and negative polarity, 5&6, 12.5 cm diameter sphere-plane gap, positive and negative polarity, 7&8, 50 cm diameter sphere plane gap, positive and negative polarity. Corona inception voltage: 9&10, point plane gap, positive and negative polarity, 11&12, 6.25 cm diameter sphere-plane gap, positive and negative polarity, 13 & 14, 12.5 cm diameter sphere-plane gap, positive and negative polarity.

The sparkover voltage for a negative point electrode is higher than that for positive polarity. The difference in the sparking voltage increases with gap length as shown in Curve *d* of Fig. 8.15. In the case of negative polarity the avalanche growth is quenched at the edge of the active region and thereafter the electrons drift in a low electric field

towards the earthed electrode. The electrons get attached to the neutral molecules forming a negative ion barrier that drifts very slowly ( $\sim$ ms) due to the low ionic mobility.

This barrier reduces the electric field in the region between the cathode and the negative ion space charge, suppressing the growth of the leader channel. Replacing the earthed electrode by a point or rod permits the propagation of vigorous positive leader channels from a point of contact of negative corona with the anode. The hatched region in Fig. 8.9 shows that a reduction of sparkover voltage with increasing gap length may be observed under certain combination of electrode geometry and atmospheric conditions such as pressure and humidity. The role of ion space charge is quite complex in these situations and a detailed analysis will not be attempted here.

## **8.10 BREAKDOWN VOLTAGES IN AIR WITH ALTERNATING VOLTAGES**

Fig. 8.16 shows the RMS value of the corona inception and breakdown voltage for various gaps in the range of gap length,  $11\mu m \leq d \leq 0.1\text{ m}$  covering a voltage range of  $0.5 \leq V \leq 90\text{ kV}^{29}$ . The gaps were irradiated with ultraviolet radiation to reduce scatter in both  $V_c$  and  $V_s$ . The scatter in observed voltages was much greater in the region where corona inception was not distinguishable from breakdown voltage. However in the region where corona inception preceded the breakdown by a significant voltage difference, the effect of irradiation was reduced, possibly due to the fact that corona provides considerable number of initiatory electrons.

The breakdown voltage of non-uniform field gaps under 50 Hz voltages is given by the empirical formulae<sup>30</sup>:

$$\begin{aligned} V_s &= 25 + 4.55 d && \text{for rod-plane gaps} \\ V_s &= 10 + 5.25 d && \text{for rod-rod gaps} \end{aligned}$$

where  $V_s$  is expressed in kilovolts and the gap length  $d$  in centimetres. For much larger gap lengths, in the range of  $1 \leq d \leq 9$  meters, the breakdown voltage follows the equation (Meek and Craggs, 1978)

$$\begin{aligned} V_s &= 0.0798 + 0.4779d - 0.0334d^2 + 0.0007d^3 && \text{rod-plane gaps} \\ V_s &= -0.990 + 0.6794d - 0.405d^2 && \text{rod-rod gaps} \end{aligned}$$

## 8.11 CONCLUDING REMARKS

In this chapter we have presented the essentials of gas discharge phenomena covering a wide range of experimental conditions. The details of the mechanisms operative in gas discharges depend upon so many parameters that an exhaustive discussion is not possible in a limited space. In addition to the dc and alternating voltages, equipments in the electrical power sector encounter transient voltages. The transient voltages may have a rise time that varies from a few ( $\sim 10$ -50) nanosecond to as long as 250  $\mu$ s. Lightning impulses have a rise time of the order of 1.5  $\mu$ s and the switching impulses have a rise time upto 300  $\mu$ s. A significant of failures of gas insulated equipments have failed in practice due to fast rising transients ( $\leq 10$  ns)<sup>31</sup>. A notable contribution has been made in predicting the breakdown in SF<sub>6</sub> under impulse conditions by Xu et. al.<sup>32</sup> and the developed model yields results that are in agreement with the measured breakdown probabilities (Fig. 8.17).

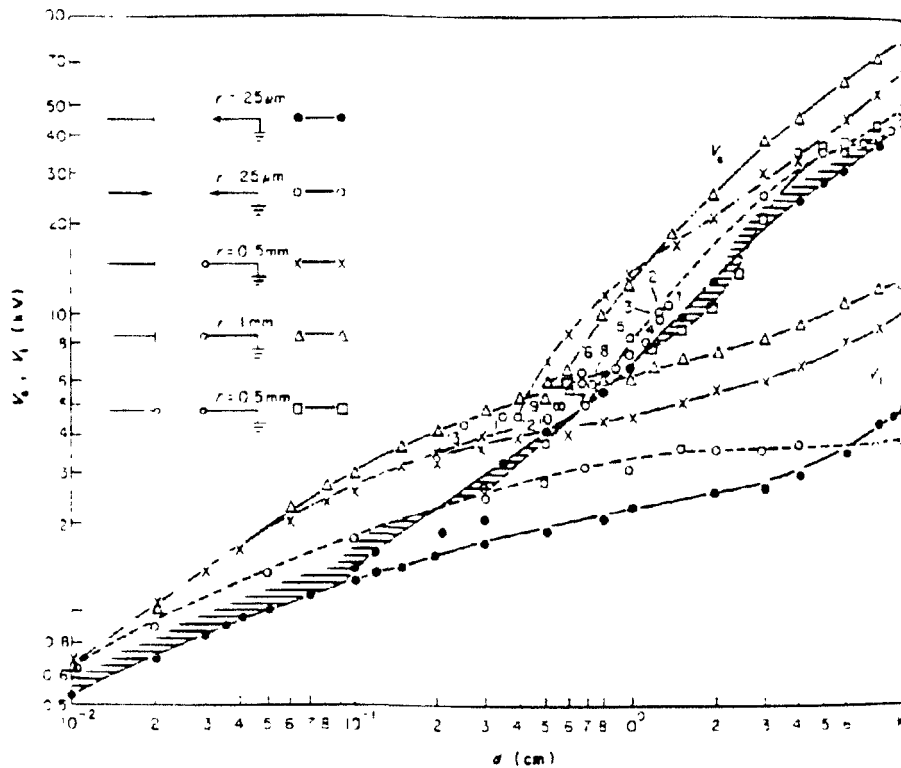


Fig. 8.16 RMS value of the 50 Hz corona inception voltage  $V_c$  and breakdown voltage  $V_s$  for different electrode radii,  $r$ , in atmospheric air. The points 1-13 refer to gaps that occur in printed circuits (Meek and Craggs, 1978, with permission of John Wiley & Sons, New York)

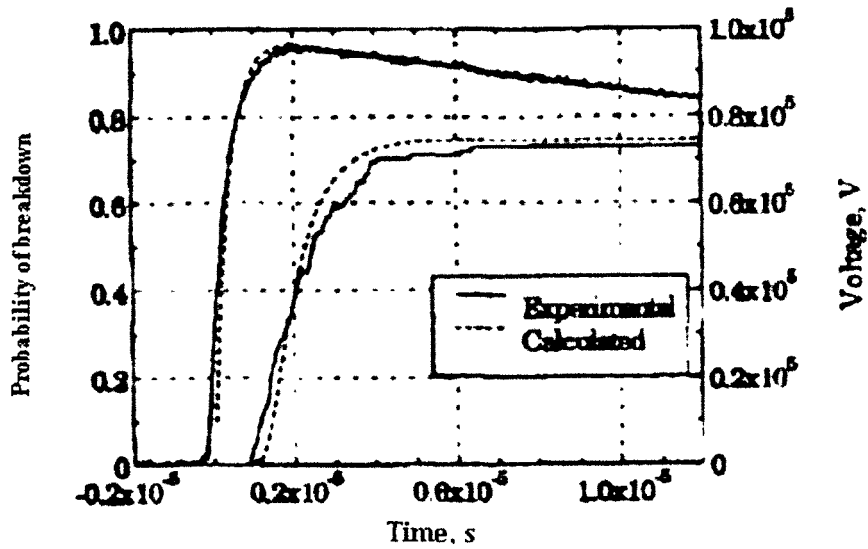


Fig. 8.17 Measured and calculated breakdown probabilities for the lightning impulse at 100 kV crest voltage along with the test voltage wave form (upper trace), (Xu et. al, 1996, ©IEEE)

The more fundamental aspects of determining the nature of corona and swarm parameters from the electron energy distribution functions will be dealt with in the next chapter.

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